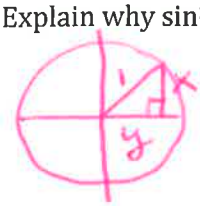
M3H Unit 5 Day 6 Pythagorean Identities – HW

1. Given $\sin^2\theta + \cos^2\theta = 1$, show algebraically how to derive $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \csc^2\theta$

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$ $1 + \cot^2\theta = \csc^2\theta$	$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ $\tan^2\theta + 1 = \sec^2\theta$

2. Explain why $\sin^2\theta + \cos^2\theta = 1$ is called a Pythagorean Identity.



in the unit circle x is $\cos\theta$ and y is $\sin\theta$
so $x^2 + y^2 = 1$ is $\sin^2\theta + \cos^2\theta = 1$.

3. Review from Math 2:

Reciprocal Identities (6)		Quotient Identities (2)
$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$	

3. Simplify the expression using Reciprocal Identities, Quotient Identities, and Pythagorean Identities:

1. **sec x cos x**

$= \frac{1}{\cos x} \cdot \cos x$

$= 1$

2. **tan2 x - sec2 x**

$= \tan^2 x - (1 + \tan^2 x)$

$= \tan^2 x - 1 - \tan^2 x$

$= -1$

3. $\frac{1-\cos^2 x}{\sin x}$

$= \frac{\sin^2 x}{\sin x}$

$= \sin x$

4. cot x sec x

$= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$

$= \frac{1}{\sin x} = \csc x$

5. cos2 x (sec2 x - 1)

$= \cos^2 x \cdot \tan^2 x$

$= \cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x}$

$= \sin^2 x$

6. $\frac{\sec^2 x - 1}{\sin^2 x}$

$= \frac{\tan^2 x}{\sin^2 x}$

$= \tan^2 x \cdot \frac{1}{\sin^2 x}$

$= \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\sin^2 x}$

$= \frac{1}{\cos^2 x} = \sec^2 x$

7. cot x sec x

same as #4

8. $\frac{\cot x}{\tan x}$

$= \frac{\cos x}{\sin x} \div \frac{\sin x}{\cos x}$

$= \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\sin x}$

$= \frac{\cos^2 x}{\sin^2 x}$

$= \cot^2 x$

or...

$= \cot\theta \cdot \frac{1}{\tan\theta}$

$= \cot\theta \cdot \cot\theta$

$= \cot^2\theta$

9. $\frac{\sin^2 x \cot x}{\cos x} \cdot \frac{1}{\cos x}$

$= \sin^2 x \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$

$= \sin x$

10. sin2 x cos2 x - cos2 x

$= \cos^2 x (\sin^2 x - 1)$

$= \cos^2 x (-\cos^2 x)$

$= -\cos^4 x$