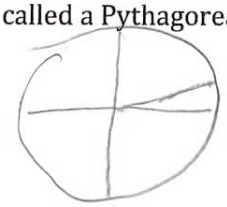
M3H Unit 5 Day 6 Pythagorean Identities – HW

1. Given $\sin^2\theta + \cos^2\theta = 1$, show algebraically how to derive $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \csc^2\theta$

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$ $1 + \cot^2\theta = \csc^2\theta$	$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ $\tan^2\theta + 1 = \sec^2\theta$

2. Explain why $\sin^2\theta + \cos^2\theta = 1$ is called a Pythagorean Identity.



$x = \cos\theta$
$y = \sin\theta$

$x^2 + y^2 = 1$

$\sin^2\theta + \cos^2\theta = 1$

3. Review from Math 2:

Reciprocal Identities (6)	Quotient Identities (2)
$\sin x = \frac{1}{\csc x}$ $\quad \csc x = \frac{1}{\sin x}$ $\cos x = \frac{1}{\sec x}$ $\quad \sec x = \frac{1}{\cos x}$ $\tan x = \frac{1}{\cot x}$ $\quad \cot x = \frac{1}{\tan x}$	$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

3. Simplify the expression using Reciprocal Identities, Quotient Identities, and Pythagorean Identities:

1. sec x cos x

$\frac{1}{\cos x} \cos x = 1$

2. tan2 x - sec2 x

$\tan^2\theta + 1 = \sec^2\theta$
$-1 \qquad -1$
$\tan^2\theta = \sec^2\theta - 1$
$\tan^2\theta - \sec^2\theta = -1$

3. $\frac{1-\cos^2 x}{\sin x}$

$= \frac{\sin^2 x}{\sin x} = \sin x$

4. cot x sec x $\quad \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} = \frac{1}{\sin x} = \csc x$

5. cos2 x (sec2 x - 1)

$= \cos^2 x \tan^2 x$

$\cos^2 \left(\frac{\sin^2}{\cos^2}\right)$

$= \sin^2 x$

6. $\frac{\sec^2 x - 1}{\sin^2 x}$ $\quad = \frac{\tan^2 \theta}{\sin^2 \theta} = \frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{\sin^2 \theta}$

$= \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{1}{\sin^2 \theta} = \frac{1}{\cos^2 \theta}$

$= \sec^2 \theta$

7. cot x sec x

$= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$

$= \frac{1}{\sin x} = \csc x$

8. $\frac{\cot x}{\tan x}$

$\frac{\frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x}} = \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\sin x}$

$= \frac{\cos^2 x}{\sin^2 x}$

$= \cot^2 x$

9. $\frac{\sin^2 x \cot x}{\cos x}$

$\frac{\sin^2 x \frac{\cos x}{\sin x}}{\cos x} = \frac{\sin x}{\cos x}$

$= \tan x$

10. sin2 x cos2 x - cos2 x

$\cos^2 \theta (\sin^2 x - 1)$

$= \cos^2 \theta (-\cos^2 \theta)$

$= -\cos^4 \theta$