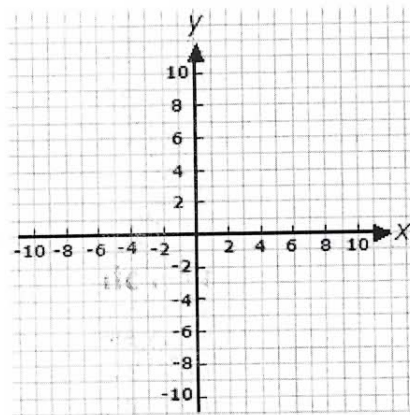


M3H Unit 5 Day 5 - Solving Logarithmic Equations

To solve a logarithmic equation:

- 1) Get all logs on one side of the equation
- 2) Use properties to condense into a single log
- 3) Isolate the log
- 4) Exponentiate
- 5) Solve for x



Extraneous Roots!

Think about the graph of $y = \log x$...

Can you take the log of a negative number?

What about $x = -2$ for $\log x$? $\log(-2x)$?

Always CHECK!

Easy examples: (Isolate/Exponentiate/solve for x)

1. $\log 6x - 3 = -4$

$$\begin{aligned} \log 6x &= -1 \\ 6x &= 10^{-1} \\ x &= \frac{1}{60} \end{aligned}$$

2. $3\log(x^2 + 1) = 5$

$$\begin{aligned} \log(x^2 + 1) &= \frac{5}{3} \\ x^2 + 1 &= 10^{5/3} \\ x^2 &= 10^{5/3} - 1 \\ x &= \pm (10^{5/3} - 1)^{1/2} \\ &= \pm 6.74 \end{aligned}$$

3. $2\ln(3x - 1) + 4 = 7$

$$2\ln(3x - 1) = 3$$

$$\begin{aligned} \ln(3x - 1) &= \frac{3}{2} \\ 3x - 1 &= e^{3/2} \\ 3x &= e^{3/2} + 1 \\ x &= (e^{3/2} + 1)/3 \\ x &\approx 1.83 \end{aligned}$$

Medium examples: (Steps 2 - 5)

1. $\log 2x - \log 3x = 7$

$$\begin{aligned} \log \frac{2x}{3x} &= 7 \\ \log \frac{2}{3} &\neq 7 \\ \text{No solution} \end{aligned}$$

2. $5\log(x + 1) + \log 4 = -2$

$$\begin{aligned} \log 4(x+1)^5 &= -2 \\ 4(x+1)^5 &= \frac{1}{100} \\ (x+1)^5 &= \frac{1}{400} \end{aligned}$$

$$\begin{aligned} x &= \left(\frac{1}{400}\right)^{1/5} - 1 \\ &\approx -0.698 \end{aligned}$$

More Complicated examples: (Steps 1 - 5)

1. $\log(4x - 5) = \log(2x - 1) + 4$

$$\begin{aligned} \log \frac{(4x-5)}{(2x-1)} &= 4 \\ \frac{4x-5}{2x-1} &= 10,000 \\ 4x-5 &= 10000(2x-1) \\ 4x-5 &= 20,000x - 10,000 \\ 9995 &= 19,996x \\ x &= .4998 \end{aligned}$$

2. $\log_5 6 + \log_5 2x^2 = \log_5 48$

$$\begin{aligned} \log_5 12x^2 &= \log_5 48 \\ \log_5 \frac{x^2}{4} &= 0 \\ \frac{x^2}{4} &= 1 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$