U5 Day 4 Properties of Logs

Logarithms can have ANY positive base b, except $b \neq 1$. Ex. $\log_2$ $\log_3$ $\log_4$ etc.

TWO Special Bases (We can put these in the calculator – round to 2 decimal places!)

Common Logs - Base 10	Natural logs - Base e

For example:

log 10 = 1 **this means:** $10^1 = 10$	**ln e =** 1 **this means:** $e^1 = e$
log 1 = 0 **this means:** $10^0 = 1$	**ln 1 =** 0 **this means:** $e^0 = 1$
log 49 ≈ 1.69 **this means:** $10^{1.69} \approx 49$	**ln 49 ≈** 3.89 **this means:** $e^{3.89} \approx$
log 4≈ .602 **this means:** $10^{.602} \approx 4$	**ln 4 ≈** 1.386 **this means:** $e^{1.386} \approx 4$

What do these two kinds of logs have in common?

1) both have zeros at x = 1 similar shape

2) Both are useful in many applications

Can we put other bases into the calculator? Only with New Operating System

Change of Base Formula

$$\log_m n = \frac{\log n}{\log m}$$

$\log_2 3.1$	$\log_5 29$	$\log_7 25$	$\log_2 3.1$
1.632	2.092	1.654	1.632

$\frac{\log 3.1}{\log 2}$ $\frac{\log 29}{\log 5}$ $\frac{\log 25}{\log 7}$ $\frac{\log 3.1}{\log 2}$

PROPERTIES of ALL logarithms, any base

Product Rule : $\log_b MN =$ $\log_b M + \log_b N$	$\log 10x$	$= \log 10 + \log x$
	$\ln 7x$	$= \ln 7 + \ln x$
Quotient Rule : $\log_b \frac{M}{N} =$ $\log_b M - \log_b N$	$\log \frac{x}{2}$	$= \log x - \log 2$
	$\ln \frac{e}{7}$	$= \ln e - \ln 7$ $= 1 - \ln 7$
Power Rule : $\log_b M^N =$ $N \log_b M$	$\log \sqrt{x}$	$= \log x^{\frac{1}{2}} = \frac{1}{2} \log x$
	$\ln x^2$	$= 2 \ln x$

Expanding Logarithms – Use properties to take a complex log apart.

$\log_5 x^2y^3$ $= \log_5 x^2 + \log_5 y^3$ $= 2\log_5 x + 3\log_5 y$	$\log_6 36x^2$ $= \log_6 36 + \log_6 x^2$ $= 2 + 2\log_6 x$	$\ln \frac{2rs}{5w}$ $= \ln 2rs - \ln 5w$ $= \ln 2 + \ln r - \ln s$ $- \ln 5 - \ln w$
$\log \frac{x}{yz}$ $= \ln x - \ln yz$ $= \ln x - (\ln y + \ln z)$ $= \ln x - \ln y - \ln z$	$\log \frac{2x^3}{z}$ $= \log 2x^3 - \log z$ $= \log 2 + \log x^3 - \log z$ $= \log 2 + 3\log x - \log z$	$\ln (3xyz)^2$ $= 2\ln(3xyz)$ $= 2(\ln 3 + \ln x + \ln y + \ln z)$ $= 2\ln 3 + 2\ln x + 2\ln y$ $+ 2\ln z$
$\ln x^2y^4$ $= \ln x^2 + \ln y^4$ $= 2\ln x + 4\ln y$	$\log_6 \frac{xy}{z}$ $= \log_6 xy - \log_6 z$ $= \log_6 x + \log_6 y - \log z$	$\ln \frac{x}{yz}$ $\ln x - \ln yz$ $= \ln x - (\ln y + \ln z)$ $= \ln x - \ln y - \ln z$

Simplifying logarithms – Use properties to write several logs with the same base as **a single log.** **(You need this skill to solve log equations.)**

$\log_2 x + \log_2 z$ $= \log_2 xz$	**$2\log_3 x - 3\log_3 y$** $\log_3 x^2 - \log_3 y^3$ $= \log_3 \frac{x^2}{y^3}$	**$4\log_2 x + 5\log_2 y$** $\log_2 x^4 + \log_2 y^5$ $= \log_2 x^4 y^5$
$\frac{1}{2}\log x + \frac{1}{4}\log 2y$ $\log x^{\frac{1}{2}} + \log(2y)^{\frac{1}{4}}$ $= \log x^{\frac{1}{2}}(2y)^{1/4}$	**$2\log x + 3\log 3y - 4\log w$** $\log x^2 + \log(3y)^3 - \log w^4$ $= \log \frac{x^2(3y)^3}{w^4}$	**$3(5\log x^3)$** $= 15\log x^3$ $\log(x^3)^{15} = \log x^{45}$
$5\log 2 + 3\log 2$ $\log 2^5 + \log 2^3$ $= \log 2^8 = \log 256$ ≈ 2.41	**$\frac{1}{2}(\ln x - \ln y)$** $= \frac{1}{2}\ln x - \frac{1}{2}\ln y$ $= \ln \frac{x^{1/2}}{y^{1/2}}$ $= \ln\left(\frac{x}{y}\right)^{\frac{1}{2}}$	**$\ln 7 - \ln 3 + \ln 6$** $\ln \frac{7(6)}{3} = \ln 14$ ≈ 2.64

$\log(27x^2y^3/w^4)$