U4 Day 2 HW - Finding the Inverse Algebraically

In these problems, determine whether f(x) is one-to-one. If it is not, restrict the domain so that $f^{-1}(x)$ is one-to-one. **Give the restricted domain and the range. Then find $f^{-1}(x)$ and write its domain and range.**

1. $f(x) = -4x - 3$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

$f(x) = -4x - 3$

$x = -4y - 3$

$x + 3 = -4y$

$y = -\frac{1}{4}x - \frac{3}{4}$

$f^{-1}(x) = -\frac{1}{4}x - \frac{3}{4}$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

2. $f(x) = \frac{5}{x-2}$

D: $(-\infty, 2) \cup (2, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

$f(x) = \frac{5}{x-2}$

$x = \frac{5}{y-2}$ $(y-2)$ $\frac{(y-2)}{1}$

$x(y-2) = 5$

$y - 2 = \frac{5}{x}$

$y = \frac{5}{x} + 2$

$f^{-1}(x) = \frac{5}{x} + 2$

D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, 2) \cup (2, \infty)$

Ex. $f(x) = \sqrt{x-1} + 3$

D: $[1, \infty)$

R: $[3, \infty)$

$f(x) = \sqrt{x-1} + 3$

$x = \sqrt{y-1} + 3$

$x - 3 = \sqrt{y-1}$

$(x-3)^2 = y - 1$

$x^2 - 6x + 9 = y - 1$

$x^2 - 6x + 10 = y$

$f^{-1}(x) = x^2 - 6x + 10$

D: $[3, \infty)$

R: $[1, \infty)$

Ex. $f(x) = -3\sqrt[3]{x+2}$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

$f(x) = -3\sqrt[3]{x+2}$

$x = -3\sqrt[3]{y+2}$

$\frac{x}{-3} = \sqrt[3]{y+2}$

$\frac{x^3}{-27} = y + 2$

$y = 2 - \frac{x^3}{27}$

$f^{-1}(x) = 2 - \frac{x^3}{27}$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

Ex. $f(x) = -2(x-3)^2 - 4$

D: $(-\infty, \infty)$

R: $(-\infty, -4]$

Must restrict To $[3, \infty)$

$f(x) = -2(x-3)^2 - 4$

$x = -2(y-3)^2 - 4$

$\frac{x+4}{-2} = (y-3)^2$

$\sqrt{\frac{x+4}{-2}} = y - 3$

$f^{-1}(x) = \sqrt{\frac{x+4}{-2}} + 3$

D: $(-\infty, -4]$

R: $[3, \infty)$

Ex. $f(x) = (x+6)^3 + 7$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

$f(x) = (x+6)^3 + 7$

$x = (y+6)^3 + 7$

$(x-7) = (y+6)^3$

$\sqrt[3]{x-7} = y+6$

$y = \sqrt[3]{x-7} - 6$

$f^{-1}(x) = \sqrt[3]{x-7} - 6$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

Ex. $f(x) = \frac{x}{x+3}$

D: $(-\infty, -3) \cup (-3, \infty)$

R: $(-\infty, 1) \cup (1, \infty)$

$f(x) = \frac{x}{x+3}$

$(y+3)\, x = \left(\frac{y}{y+3}\right)(y+3)$

$xy + 3x = y + 3$

$-y \qquad -y$

$xy - y + 3x = 3$

$-3x \qquad -3x$

$x(y-y) = -3x + 3$

$y(x-1) = -3x + 3$

$y = \frac{-3x+3}{(x-1)}$

$f^{-1}(x) = \frac{-3x+3}{x-1}$

D: $(-\infty, 1) \cup (1, \infty)$

R: $(-\infty, -3) \cup (-3, \infty)$