Solve each problem involving arithmetic/geometric sequences or series.

1. The first year a toy manufacturer introduces a new toy, its sales total $495,000. The company expects its sales to drop 10% each succeeding year. Geo, $r = .9$ $a_n = 495000(.9)^{n-1}$

   A. Find the year sales in year 6.

   $495000(.9)^{6-1} = \$292,292.55$

   B. Find the total expected sales in the first 6 years.

   $\dfrac{495000(1-.9^6)}{(1-.9)}$ $\$2,319,367.05$

2. When a pendulum swings freely, the length of its arc decreases geometrically. If the 8th arc is 20 in. and the 10th arc is 18.5 in., find the length of the 15th arc by writing a formula for the length of the pendulum swings.

   $r^2 = \dfrac{18.5}{20} \rightarrow r = .962$ $20 = a_1(.962)^{8-1}$ $a_1 = 26.23$ $26.23(.962)^{15-1} = \boxed{15.25}$

3. A bookshelf has 7 shelves of different widths. Each shelf is narrower than the shelf below it. The bottom three shelves are 36 in., 31 in., and 26 in. wide. $36 - 5(n-1) \rightarrow 41 - 5n$

   A. The shelf widths decrease by the same amount from bottom to top. What is the width of the top shelf?

   $36 - 5(7-1) = \boxed{6'}$

   B. What is the total shelf space of all seven shelves?

   $\dfrac{7}{2}(36+6) = \boxed{147'}$
   $\quad\; a_1 \quad a_n$

   C. Use summation notation to represent part B.

   $\sum\limits_{n=1}^{7}(41-5n)$

4. A seashell has chambers that are each 0.82 times the length of the enclosing chamber. The outer chamber is 32 mm around. Find the total length of the shell's spiraled chambers if there are 100 chambers? 500 chambers?

   Geo sum $\dfrac{32(1-.82^{100})}{1-.82} = \boxed{177.78}$ (getting close to $177\frac{7}{9}$) same. too small of a change

5. An embroidery pattern calls for five stitches in the first row and for three more stitches in each successive row. The 25th row, which is the last row, has 77 stitches. Find the total number of stitches in the pattern. Arith Sum

   $S = \dfrac{n}{2}(a_1 + a_n) \rightarrow \dfrac{25}{2}(5+77) = \boxed{1025}$

6. In a sequence, the first term is 8 and the 2nd term is 12. Find the 10th term if the sequence is:

   $a_1 = 8$ $d = 4$ $a_n = 8 + 4(n-1)$ $\qquad a_1 = 8$ $r = 12/8 = 1.5$

   A. Arithmetic $\qquad\qquad\qquad\qquad$ B. Geometric

   $8 + 4(10-1) = \boxed{44}$ $\qquad\qquad 8(1.5)^{10-1} \approx \boxed{307.547}$

7. Mark has saved $370 for a scooter and continues to save his weekly allowance of $10.

   A. Find the amount Mark will have saved after 7 weeks. tech. 8th term or term 1 is 380 $370 + 7 \cdot 10 = \boxed{\$440}$

   B. If the scooter costs $520, how many weeks will Mark have to save his allowance?

   $520 = 370 + 10x \rightarrow \boxed{x = 15 \text{ wks}}$

8. Beth deposits $1500 into a retirement account that pays an annual interest of 8%. Assuming Beth makes no withdrawals or deposits, $1500(1.08)^x$ or $1620(1.08)^{n-1}$ amt after yr 1

   A. How much money will she have in her account after 20 years? $1500(1.08)^{20} = \boxed{\$6991.44}$

   B. What is the first full year that she will have ten times the amount she started with?

   $15,000 = 1500(1.08)^x \rightarrow \log 10 / \log 1.08 = 29.92 \text{ yrs} \rightarrow \boxed{30 \text{ YRS}}$
   $10 = 1.08^x$

9. A side of an apartment building is shaped like a steep staircase. The windows are arranged in columns. The first column has 2 windows, the next has 4, then 6, and so on. How many windows are on the side of the apartment building if it has 15 columns? sum $\quad a_1 = 2$ $d = 2$ $n = 15$

   $\dfrac{15}{2}(2\cdot2 + 2(15-1)) = \boxed{240}$
   $n/2(2a_1 + d(n-1))$

Non Application Problems:

CHANGE

10. The first term of an arithmetic series is 123. The common difference is 12 and the sum 7098 How many terms are in the series?

$$S = \frac{a}{2}(2a_1 + d(n-1)) \rightarrow 7098 = \frac{n}{2}(246 + 12(n-1))$$
$$= \frac{a}{2}(234 + 12n) \rightarrow 7098 = 117n + 6n^2$$
$$0 = 6n^2 + 117n - 7098$$
$$n = 26$$

11. Evaluate the series: $\sum_{n=4}^{40}\left(10 - \frac{1}{2}n\right)$ $8 + 7.5 + 7 + 6.5 + \dots$

$40 - 4 + 1$

$n = 37$ $a_1 = 8$ $a_n = 10$ $\frac{37}{2}(8 - 10) = \boxed{-37}$

$n = 4$ $n = 40$