Math 3: Unit 4 Review Name: Key

Key Concepts

- Inverse Variation Word Problems (y=k/x)
- Sketching the graph of Rational Functions
- Finding Vertical Asymptotes, Horizontal Asymptotes, Holes, Domain, Range
- Simplifying Rational Expressions
- Multiplying Rational Expressions
- Dividing Rational Expressions
- Adding and Subtracting Rational Expressions with...
  - Common Denominators
  - Uncommon Denominators
- Solving Rational Equations
  - ...By Cross Multiplying
  - ...By finding the LCM
  - Checking for Extraneous Solutions

Discussion

1. In your own words, define the following:
   a. Inverse Variation → $y = k/x$ k = constant
   b. Rational Expression → $f(x) = \frac{poly}{poly}$
   c. Asymptote → Hor Vline the graph never touches
   d. Hole → when you cancel factor in num+den
   e. Domain → all possible x-values (ø V.A. or holes)
   f. Range → all possible y-values (ø H.A. or holes)
   g. Extraneous Solution → solution that doesn't work in original
   h. LCM → least common multiple
2. Describe how you would find a...
   a. Vertical Asymptote → factor completely - set each denom factor to 0
   b. Horizontal Asymptote → find degree → big bottom → y=0
      same → y = coeff.s
      big top → slant
   c. Hole → any denom factors that cancel
3. Can an x-value give you both a hole and a vertical asymptote?

   no

4. Describe the process you would do to divide rational expressions.

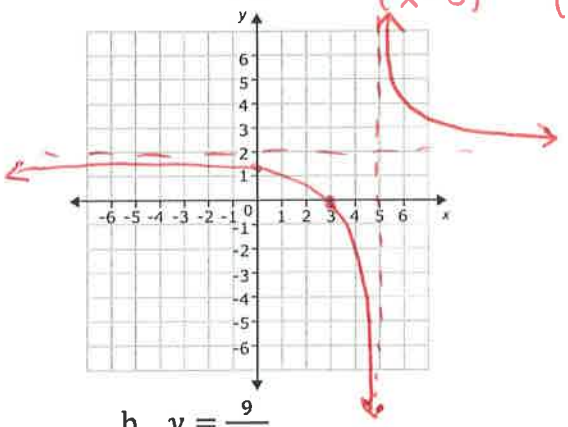
   multiply by reciprocal of 2nd fraction
   ("keep, change, flip")

Practice

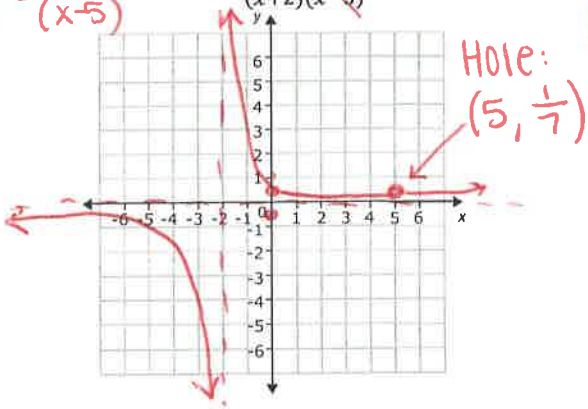
5. The length of a violin string varies inversely as the frequency of its vibrations. A violin string 14 inches long vibrates at a frequency of 450 cycles per second. Find the frequency of a 12-inch violin string. $14 = \frac{k}{450} \rightarrow k = 6300 \rightarrow 12 = \frac{6300}{x} \rightarrow x = 525$ cycles per second

6. Sketch a graph of the following:

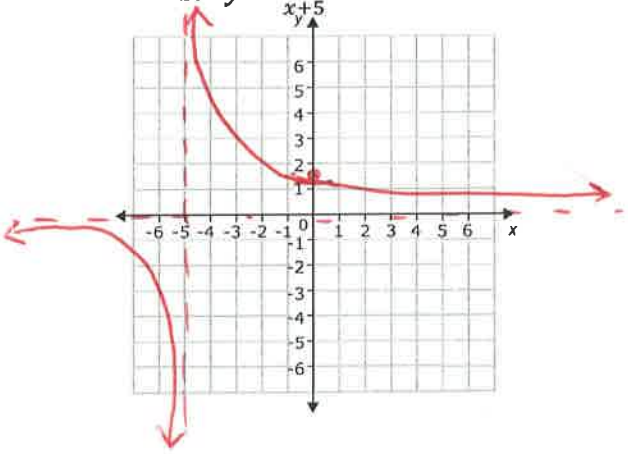
a. $y = \frac{4}{x-5} + 2 \rightarrow \frac{4+2x-10}{(x-5)} = \frac{2x-6}{(x-5)} = \frac{2(x-3)}{(x-5)}$



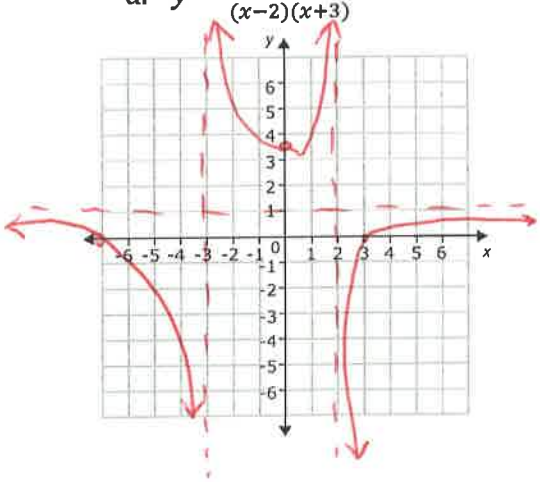
c. $y = \frac{(x-5)}{(x+2)(x-5)}$



b. $y = \frac{9}{x+5}$



d. $y = \frac{(x+7)(x-3)}{(x-2)(x+3)}$



7. Simplify the following Rational Expressions

a. $\frac{9x^2+81x}{x^3+8x^2-9x} = \frac{9x(x+9)}{x(x^2+8x-9)} = \frac{9x(x+9)}{x(x-1)(x+9)} = \boxed{\frac{9}{(x-1)}}$

b. $\frac{x^2+2x-80}{2x^3-24x^2+64x} = \frac{(x-8)(x+10)}{2x(x^2-12x+32)} = \frac{(x-8)(x+10)}{2x(x-4)(x-8)} = \boxed{\frac{x+10}{2x(x-4)}}$

c. $\frac{3r^2-39r+90}{r^2-3r-70} = \frac{3(r^2-13r+30)}{(r-10)(r+7)} = \frac{3(r-10)(r-3)}{(r-10)(r+7)} = \boxed{\frac{3(r-3)}{(r+7)}}$

$2x^2+9x+6x+27$
$x(2x+9)+3(2x+9)$

8. For the following Rational Functions, identify any Holes and Vertical and Horizontal Asymptotes. If none exist, write n/a. Then identify the Domain and the Range.

54
9 6
15

a. $y = \dfrac{3x^2+21x}{x^2+5x-14} = \dfrac{3x(x+7)}{(x+7)(x-2)}$

Hole(s)	$(-7, 7/3)$ $\frac{3(-7)}{-7-2} = \frac{-21}{-9}$
VA	$x=2$
HA	$y=3$
Domain	$(-\infty,-7)\cup(-7,2)\cup(2,\infty)$
Range	$(-\infty, 7/3)\cup(7/3, 3)\cup(3,\infty)$

b. $y = \dfrac{x-3}{2x^2+15x+27} = \dfrac{(x-3)}{(2x+9)(x+3)}$

Hole(s)	None
VA	$x=\frac{-9}{2}$ $x=-3$
HA	$y=0$
Domain	$(-\infty, \frac{-9}{2})\cup(\frac{-9}{2}, -3)\cup(-3,\infty)$
Range	$(-\infty, 0)\cup(0, \infty)$

c. $y = \dfrac{x^2-9x+20}{4x^2-12x-40} = \dfrac{(x-5)(x-4)}{4(x^2-3x-10)} = \dfrac{(x-5)(x-4)}{4(x-5)(x+2)}$

Hole(s)	$(5, 1/28)$ $\frac{5-4}{4(7)} = \frac{1}{28}$
VA	$x=-2$
HA	$y=1/4$
Domain	$(-\infty,-2)\cup(-2,5)\cup(5,\infty)$
Range	$(-\infty, 1/28)\cup(1/28, 1/4)\cup(1/4,\infty)$

d. $y = \dfrac{x+7}{(3x-2)(x+7)}$

Hole(s)	$(-7, -1/23)$ $\frac{1}{-23}$
VA	$x=2/3$
HA	$y=0$
Domain	$(-\infty,-7)\cup(-7, 2/3)\cup(2/3,\infty)$
Range	$(-\infty, -1/23)\cup(-1/23, 0)\cup(0,\infty)$

9. Perform the Operations below:

a. $\dfrac{1}{p-9}\cdot\dfrac{p^2+6p-27}{p+9} = \dfrac{(p+9)(p-3)}{(p-9)(p+9)} = \boxed{\dfrac{p-3}{p-9}}$

b. $\dfrac{v-7}{v+6}\cdot\dfrac{10v+60}{v-7} = \dfrac{(v-7)10(v+6)}{(v+6)(v-7)} = \boxed{10}$

c. $\dfrac{16x-56}{8}\div\dfrac{8x-28}{4} = \dfrac{8(2x-7)}{8}\cdot\dfrac{4}{4(2x-7)} = \boxed{1}$

d. $\dfrac{a-4}{a^2-2a-8}\div\dfrac{1}{a-5} = \dfrac{(a-4)}{(a-4)(a+2)}\cdot\dfrac{(a-5)}{1} = \boxed{\dfrac{a-5}{a+2}}$

e. $\dfrac{2x}{3x+3}-\dfrac{2}{x+5} = \dfrac{2x(x+5)}{3(x+1)(x+5)}-\dfrac{2(3x+3)}{3(x+1)(x+5)} = \dfrac{2x^2+10x-6x-6}{3(x+1)(x+5)} = \dfrac{2x^2+4x-6}{3(x+1)(x+5)}$

$\dfrac{2(x^2+2x-3)}{3(x+1)(x+5)}$

$\dfrac{2(x-1)(x+3)}{3(x+1)(x+5)}$

f. $\dfrac{3(x-8)}{x+7}+\dfrac{4(x+7)}{x-8} = \dfrac{3x-24+4x+28}{(x+7)(x-8)} = \dfrac{7x+4}{(x+7)(x-8)}$

g. $\dfrac{r+6}{3r-6}+\dfrac{r+1}{3r-6} = \dfrac{2r+7}{3r-6}$

h. $\dfrac{x+y}{18xy}-\dfrac{6x+y}{18xy}$

$\dfrac{-5x}{18xy} = \boxed{\dfrac{-5}{18y}}$

10. Solve the following equations:

a) $\frac{1}{5x} = \frac{1}{9x}$ $5x = 9x$ $x = 0$ extra!

NO Solution

b) $\frac{4}{2x-3} = \frac{x}{5}$ $20 = 2x^2 - 3x$

$0 = 2x^2 - 3x - 20$

$0 = (2x+5)(x-4)$

$x = \frac{-5}{2}$ $x = 4$

c) $\frac{3}{2x} - \frac{5}{3x} = 2$

$\frac{9}{6x} - \frac{10}{6x} = 2 \rightarrow \frac{-1}{6x} = \frac{2}{1} \rightarrow 12x = -1$

$x = \frac{-1}{12}$

challenge: try it

d) $\frac{1}{2} \leq \frac{1}{x+3} + \frac{1}{x}$ $x \leq 3$ $x \leq -2$

e) $\frac{3}{x} = \frac{12}{x+7}$ $3x + 21 = 12x$ $21 = 9x$

$7/3 = x$

f) $\frac{2}{y} + \frac{1}{2} = \frac{5}{2y}$ $\frac{4+y}{2y} = \frac{5}{2y}$ $4 + y = 5$

$y = 1$

g) $\frac{10}{6x+7} = \frac{6}{2x+9}$ $20x + 90 = 36x + 42$

$48 = 16x$

$3 = x$

h) $\frac{2}{x+2} - \frac{1}{x} = \frac{-4}{x(x+2)}$ $(x+2)$

$2x - x - 2 = -4$

$x = -2$

extra!

NO Solution

i) $\frac{3}{x+5} + \frac{-2}{-5+x} = \frac{-4}{x^2-25}$

$3x - 15 - 2x - 10 = -4$

$x - 25 = -4$

$x = 21$

j) $\frac{10}{2y+8} - \frac{7y+8}{y^2-16} = \frac{-8}{2y-8}$

trial

$2(y+4)$ $(y+4)(y-4)$ $2(y-4)$

$10(y-4) - 14y - 16 = -8(y+4)$

$10y - 40 - 14y - 16 = -8y - 32$

$-4y - 56 = -8y - 32$

$4y = 24$

$y = 6$