

## U2 Day 5 - Polynomial and Synthetic Division

**SWBAT** \_\_\_\_\_

**Long Division... remember this process from elementary school?** \_\_\_\_\_

$$\begin{array}{r}
 \text{Quotient} \leftarrow 232 \overset{7}{5} \\
 \text{Divisor} \rightarrow 15 \overline{) 3487} \\
 \underline{30} \quad \leftarrow \text{Dividend} \\
 48 \\
 \underline{45} \\
 37 \\
 \underline{30} \\
 7 \leftarrow \text{Remainder}
 \end{array}$$

Know these terms!

$$\frac{x^2}{x} = x$$

**Use the same process with polynomials:**

$$\begin{array}{r}
 x+2 \quad R \quad 3 \\
 x+1 \overline{) x^2+3x+5} \\
 \underline{-(x^2+x)} \\
 2x+5 \\
 \underline{-(2x+2)} \\
 3
 \end{array}$$

1. Take the first term  $x^2$  and divide by the first term  $x$
  2. Multiply the first part of the answer by the divisor.
  3. Draw the line, change the signs, and combine the terms.
  4. Bring down the next term.
- Repeat.

$$\frac{2x^4}{2x} = x^3$$

$  \begin{array}{r}  2x-4 \quad R \quad 14 \\  x+2 \overline{) 2x^2+0x+6} \quad (\text{Use } +0x \text{ if there is a missing term!!}) \\  \underline{-(2x^2+4x)} \\  -4x+6 \\  \underline{-(-4x-8)} \\  14  \end{array}  $ <p>Is <math>(x+2)</math> a factor of the polynomial? Yes <input type="radio"/> No <input checked="" type="radio"/></p> <p>Write the factored form: _____</p>	$  \begin{array}{r}  x^3+2x^2-x \\  2x-1 \overline{) 2x^4+3x^3-4x^2+x+1} \\  \underline{-(2x^4-x^3)} \\  4x^3-4x^2 \\  \underline{-(4x^3-2x^2)} \\  -2x^2+x \\  \underline{-(-2x^2+x)} \\  -1  \end{array}  $ <p>Is <math>(2x-1)</math> a factor of the polynomial? Yes <input type="radio"/> No <input checked="" type="radio"/></p>
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<p><b>Divide</b> <math>x^4 - 10x^2 + 2x + 3</math> <b>by</b> <math>(x-3)</math></p> $  \begin{array}{r}  x^3+3x^2-x-1 \\  (x-3) \overline{) x^4+0x^3-10x^2+2x+3} \\  \underline{-(x^4-3x^3)} \\  3x^3-10x^2 \\  \underline{-(3x^3-9x^2)} \\  -x^2+2x \\  \underline{-(-x^2+3x)} \\  -x+3 \\  \underline{-(-x+3)} \\  0  \end{array}  $ <p><math>\frac{3x^3}{x} = 3x^2</math>  <math>\frac{-x^2}{x} = -x</math></p>	<p>What is the remainder? <u>0</u></p> <p>This means that <math>(x-3)</math> is a FACTOR of the polynomial.</p> $\frac{-x}{x} = -1$ <p>What is the quotient polynomial? <u><math>x^3+3x^2-x-1</math></u></p>
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Synthetic Division (x-3)

The divisor is **always** in the form Linear factor. The **synthetic** divisor is 0 (INSIDE 3)

You **must** use a zero for each \_\_\_\_\_

ex. **Divide**  $x^4 - 10x^2 + 2x + 3$  **by**  $(x - 3)$  (Use 3 for the divisor)

(need a zero?)  $x^4 + 0x^3 - 10x^2 + 2x + 3$

**Form for Synthetic Division**

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -10 & 2 & 3 \\ & \downarrow & 3 & 9 & -3 & -3 \\ \hline & 1 & 3 & -1 & -1 & 0 \end{array}$$
 Place holder  
 ← Remainder  
 Quotient →

1. Set up the synthetic division form.
  2. Bring down the first # below the line
  3. Multiply by 3 and put the answer above the line
  4. Add
- Repeat

The quotient is:  $x^3 + 3x^2 - x - 1$



The first term of the quotient is always **one degree less than the degree of the** dividend

Is your answer the same as the last long division problem? \_\_\_\_\_ :-)

**Try this:**

<p><math>(x^3 - 28x - 48)</math> divided by <math>(x + 4)</math></p> $\begin{array}{r rrrr} -4 & 1 & 0 & -28 & -48 \\ & \downarrow & -4 & 16 & 48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$ <p>The quotient is <u><math>x^2 - 4x - 12</math></u></p>	<p><math>(x^5 - 1)</math> divided by <math>(x - 1)</math></p> $\begin{array}{r rrrrrrr} 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ & \downarrow & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 0 \end{array}$ <p>The quotient is <u><math>x^4 + x^3 + x^2 + x + 1</math></u></p>
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**Remainder Theorem**

Ex. For  $f(x) = 3x^2 - 5x + 7$  find  $f(-2)$

Substitute  $3(-2)^2 - 5(-2) + 7 = 29$  so  $f(-2) = 29$

**The Remainder Theorem (shortcut):**

Ex: 
$$\begin{array}{r|rrrr} -2 & 3 & -5 & 7 \\ & \downarrow & -6 & 22 \\ \hline & 3 & -11 & 29 \end{array}$$

The remainder: 29 is the same as 29  
 $= -2(-2)^5 + 4(-2)^3 - 5(-2)^2 + 7(-2) - 11$

Use the Remainder Theorem below to find  $f(-3)$  if  $f(x) = -2x^5 + 4x^3 - 5x^2 + 7x - 11$   $f(-3) =$  \_\_\_\_\_

$$\begin{array}{r|rrrrrr} -3 & -2 & 0 & 4 & -5 & 7 & -11 \\ & \downarrow & 6 & -18 & 42 & -111 & 312 \\ \hline & -2 & 6 & -14 & 37 & -104 & 301 \end{array}$$

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HW: Homework WS

**The Remainder Theorem:**

If a polynomial  $P(x)$  is divided by  $(x - c)$ , the remainder in synthetic division by  $c$  is equal to  $P(c)$ .

Use the Remainder Theorem to find  $f(4)$  if  $f(x) = 3x^4 - 4x^3 + 6x + 1$ .

**In Closing...**

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$$\begin{array}{r|rrrrr} 4 & 3 & -4 & 0 & 6 & 1 \\ & & 12 & 32 & 128 & 536 \\ \hline & 3 & 8 & 32 & 134 & 537 \end{array}$$

$$3(4)^4 - 4(4)^3 + 6(4) + 1 = 537$$