

Problem 5: Situation for a "Fungi"



A biologist is developing two new strains of bacteria. Each sample of Type I bacteria produces 4 new viable bacteria and each sample of Type II produces 3 new viable bacteria. Altogether, at least 240 new viable bacteria must be produced. At least 30, but no more than 60, of the original samples must be Type I. No more than 70 of the samples can be type II. A sample of Type I cost \$7 and a sample of Type II costs \$3. How many samples of each should be used to minimize the cost? What is the minimum cost?

Variables (in words): $x =$ # Type 1 samples $y =$ # Type 2 samples

Constraints:

$$\begin{aligned} 4x + 3y &\geq 240 & (0, 80), (60, 0) \\ 30 &\leq x \leq 60 \\ y &\leq 70 \end{aligned}$$

$$\begin{aligned} 4(30) + 3y &= 240 \\ 120 + 3y &= 240 \\ -120 & \quad -120 \\ 3y &= 120 \\ y &= 40 \end{aligned}$$

Objective Function:

$$C(x, y) = 7x + 3y$$

Vertices of Feasible Region:

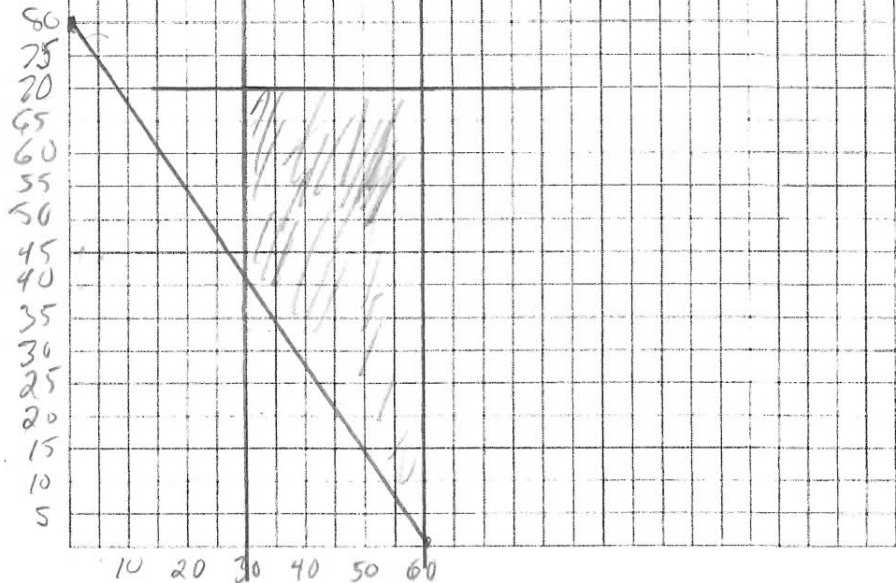
$$(30, 70), (60, 70), (60, 0), (30, 40)$$

Ordered Pair of Optimal Solution:

$$(30, 40)$$

Minimum Cost:

$$C(30, 40) = 330$$



$$\begin{aligned} C(30, 70) &= 7(30) + 3(70) \\ &= 420 \end{aligned}$$

$$\begin{aligned} C(60, 70) &= 7(60) + 3(70) \\ &= 630 \end{aligned}$$

$$\begin{aligned} C(60, 0) &= 7(60) + 3(0) \\ &= 420 \end{aligned}$$

$$\begin{aligned} C(30, 40) &= 7(30) + 3(40) \\ &= 330 \end{aligned}$$

Problem 6: "Fat"astic Meals

A school dietitian wants to prepare a meal of meat and vegetables that have the lowest possible fat and that meet the Food and Drug Administration recommended daily allowances (RDA) of iron and protein. The RDA minimums are 24 milligrams of iron and 50 grams of protein. Each serving of meat contains 10 grams of protein, 4 milligrams of iron, and 5 grams of fat. Each serving of vegetables contains 5 grams of protein, 6 milligrams of iron, and 3 grams of fat. Write an objective function for the number of grams of fat, and find the minimum number of grams of fat. What is the minimum number of grams of fat?

Variables (in words): $x =$ # serving of meat $y =$ # servings of veggies

$$4(4.5) + 6y = 24$$

$$18 + 6y = 24$$

$$6y = 6$$

$$y = 1$$

$$6y = 6$$

$$y = 1$$

$$10x + 5y = 50$$

$$-10x$$

$$5y = -10x + 50$$

$$y = -2x + 10$$

Constraints:

$$10x + 5y \geq 50$$

$$4x + 6y \geq 24$$

$$(0, 10), (5, 0)$$

$$(0, 4), (6, 0)$$

$$4x + 6(-2x + 10) = 24$$

$$4x - 12x + 60 = 24$$

$$-8x + 60 = 24$$

$$-8x = -36$$

$$x = 4.5$$

Objective Function:

$$F(x, y) = 5x + 3y$$

Vertices of Feasible Region:

$$(0, 10), (6, 0), (4.5, 1)$$

Ordered Pair of Optimal Solution:

$$(4.5, 1)$$

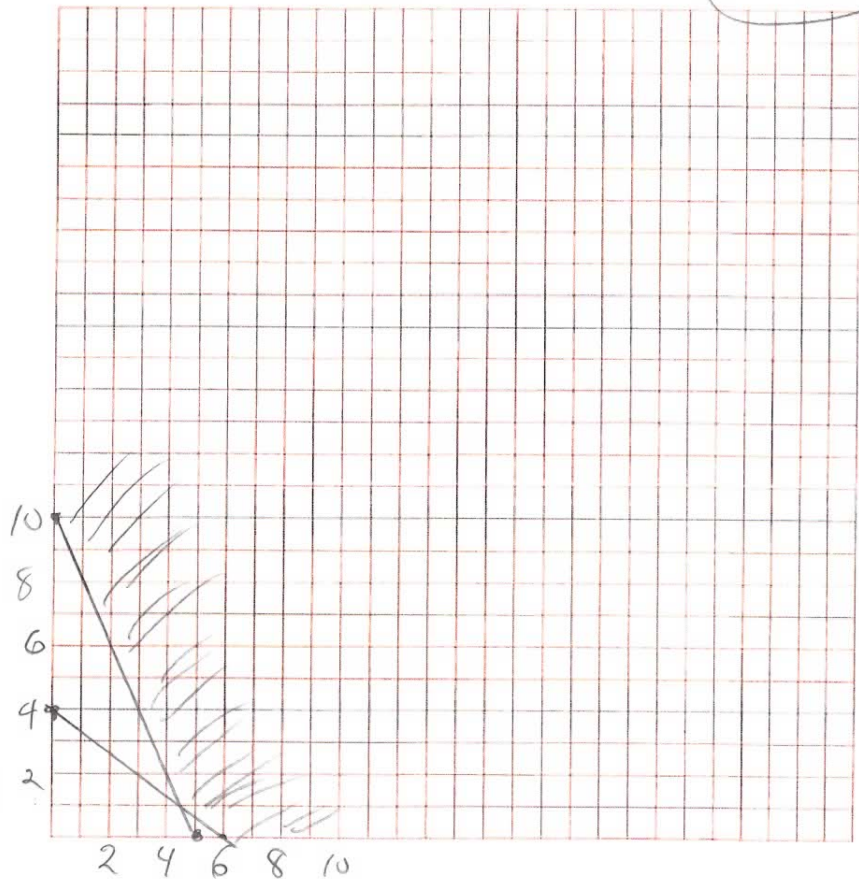
Minimum Fat Grams:

$$F(4.5, 1) = 25.5$$

$$F(0, 10) = 5(0) + 3(10) = 30$$

$$F(6, 0) = 5(6) + 3(0) = 30$$

$$F(4.5, 1) = 5(4.5) + 3(1) = 25.5$$



Problem 7: Be a Doll and Solve This



One of the dolls that Barbiehoo-R-Us manufactures is the *Talking Tommy*. Another doll without this talking mechanism is called *Silent Sally*. In one hour, the company can produce 8 *Talking Tommy* dolls or 20 *Silent Sally* dolls. Because of the demand, the company knows that it must produce at least twice as many *Talking Tommy* dolls as the *Silent Sally* dolls. The company spends no more than 48 hours per week making these two dolls. The profit on each *Talking Tommy* is \$3.00 and the profit on each *Silent Sally* is \$7.50. How many of each doll should be produced to maximize profit each week? What is this profit?

Variables (in words): $x =$ # *Talking Tommy* $y =$ # *Silent Sally*

Constraints: $x \geq 2y$ or $y \leq \frac{1}{2}x$
 $\frac{1}{8}x + \frac{1}{20}y \leq 48$ $(0, 960), (384, 0)$
 $x \geq 0, y \geq 0$
 $y = \frac{1}{2}(320) = (160)$

$$\begin{aligned}\frac{1}{8}x + \frac{1}{20}\left(\frac{1}{2}x\right) &= 48 \\ \frac{1}{8}x + \frac{1}{40}x &= 48 \\ \frac{5}{40}x + \frac{1}{40}x &= 48 \\ \frac{6}{40}x &= 48 \\ x &= 320\end{aligned}$$

Objective Function:

$$P(x, y) = 3x + 7.5y$$

Vertices: of Feasible Region:

$$(0, 0), (320, 160), (384, 0)$$

Ordered Pair of Optimal Solution:

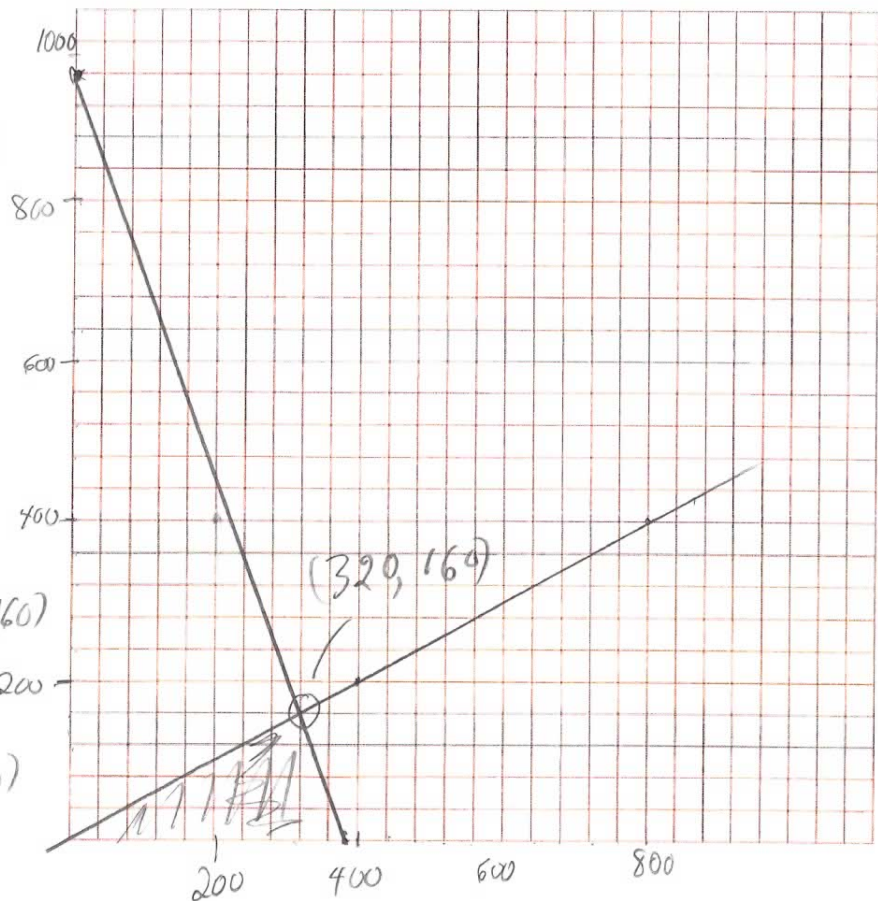
$$(320, 160)$$

Maximum Profit:

$$P(320, 160) = 1152$$

$$\begin{aligned}P(320, 160) &= 3(320) + 7.5(160) \\ &= \$2160\end{aligned}$$

$$\begin{aligned}P(384, 0) &= 3(384) + 7.5(0) \\ &= \$1152\end{aligned}$$



Problem 8: Spike for a Goal



A sporting goods manufacturer Soc-It-To-Ya makes a profit of \$5 on soccer balls and a profit of \$4 on volleyballs. Cutting requires 2 hours to make 75 soccer balls and 3 hours to make 60 volleyballs. Sewing needs 3 hours to make 75 soccer balls and 2 hours to make 60 volleyballs. Cutting has 500 hours available and Sewing has 450 hours available. How many soccer balls and volleyballs should be made to maximize profit? What is this profit?

Variables (in words): $x =$ # Soccer balls $y =$ # volleyballs

Constraints:

$$\frac{2}{75}x + \frac{3}{60}y \leq 500$$

$$\frac{3}{75}x + \frac{2}{60}y \leq 450$$

$$x \geq 0, y \geq 0$$

Objective Function:

$$P(x, y) = 5x + 4y$$

Vertices: of Feasible Region:

$$(0, 0), (5250, 7200)$$

$$(0, 10000), (11250, 0)$$

Ordered Pair of Optimal Solution:

$$(0, 0), (0, 10000)$$

$$(5250, 7200), (11250, 0)$$

Maximum Profit:

$$P(0, 10000) = 5(0) + 4(10000) = 40,000$$

$$P(5250, 7200) = 5(5250) + 4(7200) = 55,050$$

$$P(11,250, 0) = 5(11250) + 4(0) = 56,250$$

$$y = -\frac{8}{15}x + 10,000$$

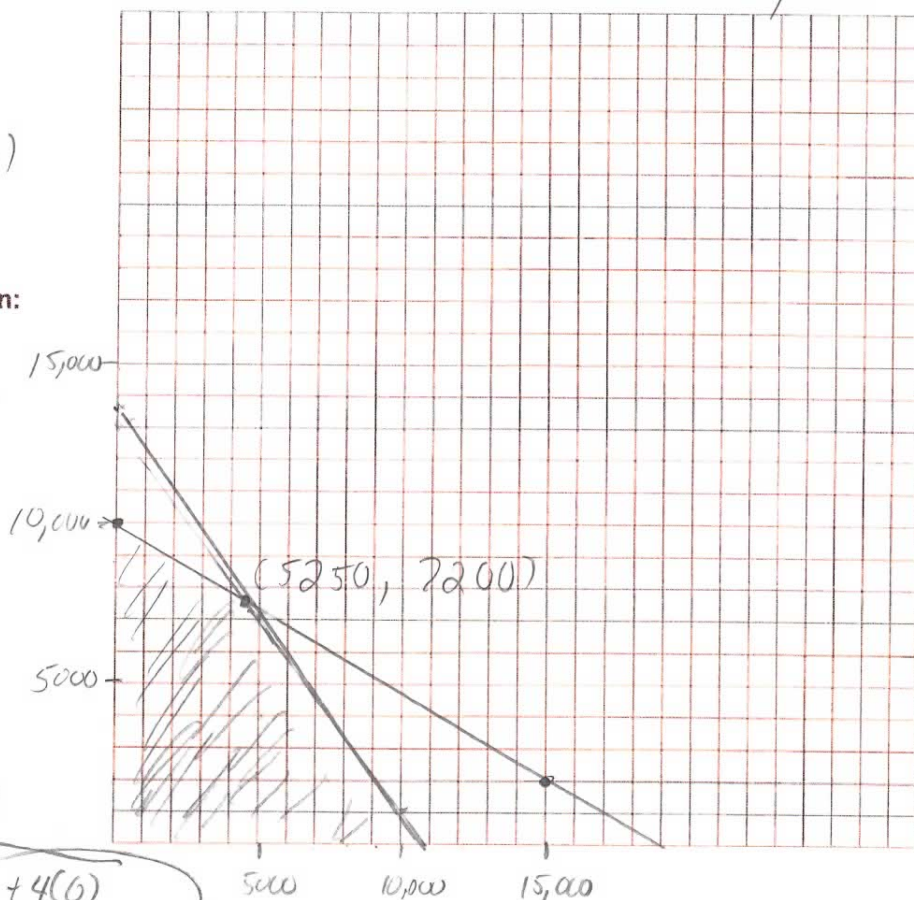
$$y = -\frac{6}{5}x + 13,500$$

$$-\frac{8}{15}x + 10,000 = -\frac{6}{5}x + 13,500$$

$$\frac{2}{3}x = 3,500$$

$$x = 5250$$

$$y = 7200$$



Hint: You will need to think about how long it takes to make ONE soccer ball or volleyball (Fractions!)

Problem 9 (Released Test Question)

Paul sells chocolate chip cookies and peanut butter cookies.

- Baking a batch of chocolate chip cookies takes 1.75 cups of flour and 2 eggs.
- Baking a batch of peanut butter cookies takes 1.25 cups of flour and 1 egg.
- Paul has 10 cups of flour and 12 eggs.
- He makes \$4 profit per batch of chocolate chip cookies.
- He makes \$2 profit per batch of peanut butter cookies.

How many batches of peanut butter cookies should Paul make to maximize his profit?

A.1

B.2

C.5

D.8

Variables (in words): $x =$ # batches of chocolate chip

$y =$ # batches of peanut butter

Constraints:

$$\begin{aligned} 1.75x + 1.25y &\leq 10 \\ 2x + y &\leq 12 \quad (0, 12), (6, 0) \\ x &\geq 0, y \geq 0 \end{aligned}$$

$$\begin{aligned} 1.25y &= -1.75x + 10 \\ y &= -\frac{7}{5}x + 8 \end{aligned}$$

Objective Function:

$$P(x, y) = 4x + 2y$$

Vertices of Feasible Region:

$$(0, 8), (7.6, 0), (0, 0)$$

Ordered Pair of Optimal Solution:

$$(7.6, 0)$$

Minimum Cost:

$$P(0, 0) = 0$$

$$P(0, 8) = 4(0) + 2(8) = 16$$

$$\begin{aligned} P(7.6, 0) &= 4(7.6) + 2(0) \\ &= 30.4 \end{aligned}$$

