

Warm Up

- 1) What is margin of error for 222 students?
- 2) Find the sample size needed for 2.4% margin of error.
- 3) How many would you need to sample to cut your margin of error in half? (Refer to #2)

Answers

1. Margin of Error = $\frac{1}{\sqrt{222}} = \pm .067$

2. $.024 = \frac{1}{\sqrt{n}}$; $\sqrt{n} = \frac{1}{.024}$; $n = 1736.1 = 1737$

3. $.012 = \frac{1}{\sqrt{n}}$; $\sqrt{n} = \frac{1}{.012}$; $n = 6944.4 = 6945$

n is always rounded up

A little more review

Identify the population and the sample:

a) A survey of 1353 American households found that 18% of the households own a computer.

b) A recent survey of 2625 elementary school children found that 28% of the children could be classified obese.

Answers

- a) Population – All American households
Sample – 1353 American households

- b) Population – All elementary school children
Sample – 2625 elementary school children

Probability Distribution

- A **probability distribution** provides the possible values of the random variable and their corresponding probabilities.
- A probability distribution can be in the form of a **table, graph, words** or mathematical **formula**.

List of possible values

Probability of each value

x	0	1	2
$P(X=x)$	$1/4$	$1/2$	$1/4$

Example: Consider tossing a fair coin 3 times.
Define X = the number of heads obtained

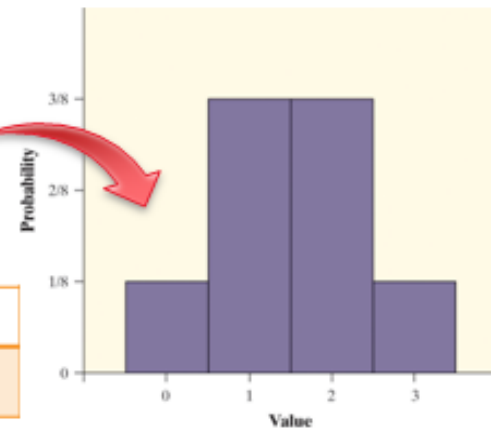
$X = 0$: TTT

$X = 1$: HTT THT TTH

$X = 2$: HHT HTH THH

$X = 3$: HHH

Value	0	1	2	3
Probability	$1/8$	$3/8$	$3/8$	$1/8$



Probability Distributions

Definitions

a) Random variable is a variable whose values are determined by chance.

b) Discrete Probability distribution consists of the values a random variable can assume and the corresponding probabilities of the values.

TWO Requirements for a Probability distribution

a) All probabilities must be between 0 and 1

b) The sum of the probabilities must add up to 1

Is the following a probability distribution?

x	$P(X = x)$
0	0.16
1	0.18
2	0.22
3	0.10
4	0.3
5	0.01

Answer:

$$0.16 + 0.18 + 0.22 + 0.10 + 0.3 + 0.01 = 0.97 < 1, \text{ Not a probability distribution}$$

Is the following a probability distribution?

x	$P(X = x)$
0	0.16
1	0.18
2	0.22
3	0.10
4	0.3
5	0.04

Answer:

$$0.16 + 0.18 + 0.22 + 0.10 + 0.3 + 0.04 = 1$$

It is a probability distribution

Expected Value

$$E(x) = \sum_{i=1}^N x_i p_i$$

$E(x)$ = expected value for x

X : value

P : probability of x happening

i : counter

N : Maximum number for I

Σ : sum of all weighted averages.

The expected value is the mean or weighted average of all the probabilities in the distribution of the x values.

It is the most likely value, on average.

EXAMPLE *The Mean of a Discrete Random Variable*

Compute the mean of the following probability distribution which represents the number of DVDs a person rents from a video store during a single visit.

x	$P(X = x)$
0	0.06
1	0.58
2	0.22
3	0.10
4	0.03
5	0.01

Answer

$$E(X) = 0(0.06) + 1(0.58) + 2(0.22) + 3(0.10) + 4(0.03) + 5(0.01) \\ = 1.49$$

or the mean # of DVDs rented is 1.49

EXAMPLE

Insurance Policy

A term life insurance policy will pay a beneficiary a certain sum of money upon the death of the policy holder. These policies have premiums that must be paid annually.

Suppose a life insurance company sells a \$250,000 one year term life insurance policy to a 49-year-old female for \$520. According to the National Vital Statistics Report, Vol. 47, No. 28, the probability the female will survive the year is 0.99791. Compute the expected value of this policy to the insurance company.

Find each x and its probability...

Possible Outcomes	$X =$ Income to Insurance Company	$P(X = x)$
She Lives	\$520	.99791
She Dies	\$520 - \$250,000	1 - .99791

$$E(x) = (520 - 250,000) * (1 - 0.99791) + 520 * 0.99791$$

= -2.5 or They are most likely to lose \$2.50

Example Investment

A local club plans to invest \$10000 to host a baseball game. They expect to sell tickets worth \$15000. But if it rains on the day of game, they won't sell any tickets and the club will lose all the money invested. If the weather forecast for the day of game is 20% possibility of rain, is this a good investment?

Possible Outcomes	X Gain/Loss	P(X = x)
Rain	-10,000	.20
No Rain	5,000	.80

Expected Value = $5000(.80) + (-10000)(.2) = 2000$

This would be a good investment.

Example Profit/Loss

A company makes electronic gadgets. One out of every 50 gadgets is faulty, but the company doesn't know which ones are faulty until a buyer complains. Suppose the company makes a \$3 profit on the sale of any working gadget, but suffers a loss of \$80 for every faulty gadget because they have to repair the unit. Check whether the company can expect a profit in the long term.

Possible Outcomes	X Gain/Loss	P(X - x)
Defective	-80	1/50
Not Defective	3	49/50

Expected Value = $(-80)(1/50) + 3(49/50) = 1.34$

Since the Expected Value is positive the operation is profitable

A Statistically Fair Game

A statistically Fair Game is a competition where both participants will break even over the long run. If a game is fair the expected value for both players will be zero.

If the expected value is not zero, the player with a positive expected value will win over the long run.

If a player pays a fee to play a game, his or her expected value must equal the fee in order for the game to be Fair.

Example Unfair Card Game

You pay \$3.00 to play. The dealer deals you one card from a standard deck. If it is a spade, you get \$10. If it is anything else, you lose your money. Is this game fair?

Expected value = $10(1/4) = \$2.50$

$\$2.50 - \$3.00 = -.50$

The game is not Fair

Example What is a Fair Fee for this Game?

A player rolls a die and receives the number of dollars equal to the number on the die EXCEPT when the die shows a 6. If a 6 is rolled, the player loses \$6. If the game is to be fair, what should be the cost to play?

X	1	2	3	4	5	-6
P(X = x)	1/6	1/6	1/6	1/6	1/6	1/6

$$\begin{aligned}\text{Expected Value} &= (1/6 + 2/6 + 3/6 + 4/6 + 5/6 - 6/6) \\ &= 9/6 = \$1.50\end{aligned}$$

If you pay \$1.50 to play this game it is Fair