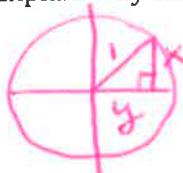


## M3H Unit 5 Day 6 Pythagorean Identities - HW

1. Given  $\sin^2\theta + \cos^2\theta = 1$ , show algebraically how to derive  $1 + \tan^2\theta = \sec^2\theta$  and  $1 + \cot^2\theta = \csc^2\theta$

$\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$ $1 + \cot^2\theta = \csc^2\theta$	$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ $\tan^2\theta + 1 = \sec^2\theta$
---	--

2. Explain why  $\sin^2\theta + \cos^2\theta = 1$  is called a Pythagorean Identity.

 in the unit circle  $x$  is  $\cos\theta$  and  $y$  is  $\sin\theta$   
 $\text{so } x^2 + y^2 = 1 \text{ is } \sin^2\theta + \cos^2\theta = 1$ .

3. Review from Math 2:

Reciprocal Identities (6)	Quotient Identities (2)
$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
	$\cot\theta = \frac{\cos\theta}{\sin\theta}$

3. Simplify the expression using Reciprocal Identities, Quotient Identities, and Pythagorean Identities:

1.  $\sec x \cos x$

$$\begin{aligned}
 &= \frac{1}{\cos x} \cdot \cos x \\
 &= 1
 \end{aligned}$$

2.  $\tan^2 x - \sec^2 x$

$$\begin{aligned}
 &= \tan^2 x - (1 + \tan^2 x) \\
 &= \tan^2 x - 1 - \tan^2 x \\
 &= -1
 \end{aligned}$$

$$3. \frac{1 - \cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x}{\sin x}$$

$$= \sin x$$

$$4. \cot x \sec x$$

$$= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$$

$$= \frac{1}{\sin x} = \boxed{\csc x}$$

$$5. \cos^2 x (\sec^2 x - 1)$$

$$= \cos^2 x \cdot \tan^2 x$$

$$= \cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x}$$

$$= \sin^2 x$$

$$6. \frac{\sec^2 x - 1}{\sin^2 x}$$

$$= \frac{\tan^2 x}{\sin^2 x}$$

$$= \tan^2 x \cdot \frac{1}{\sin^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\sin^2 x}$$

$$= \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

7.  $\cot x \sec x$

Same as #4

$$8. \frac{\cot x}{\tan x}$$

$$= \frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x}$$

$$= \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\sin x}$$

$$= \frac{\cos^2 x}{\sin^2 x}$$

$$= \cot^2 x$$

or...

$$\left. \begin{aligned} &= \cot \theta \cdot \frac{1}{\tan \theta} \\ &= \cot \theta \cdot \cot \theta \\ &= \cot^2 \theta \end{aligned} \right\}$$

$$9. \frac{\sin^2 x \cot x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \sin^2 x \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$$

$$= \sin x$$

$$10. \sin^2 x \cos^2 x - \cos^2 x$$

$$= \cos^2 x (\sin^2 x - 1)$$

$$= \cos^2 x (-\cos^2 x)$$

$$= -\cos^4 x$$