Warm Up:

Sketch the graphs of $y = x^2$, $y = x^3$, $y = x$, $y = 1/x$, $y = \sqrt{x}$, $y = \sqrt[3]{x}$ **(Present - St Eng 5pts.)**

SWBAT: 1) Sketch the graph of the inverse of a function (Reflection over y = x, switch x and y)

2) Give its Domain, Range, and End Behavior

3) Restrict the domain of a function so that its inverse is a function

I Graph of an Inverse Function - Partner Investigation

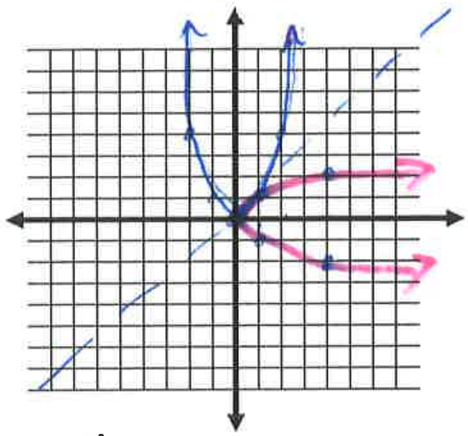
Graph: $y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

Graph: the inverse

x f⁻¹(x)

y	x
4	-2
1	-1
0	0
1	1
4	2



Is the graph a function? yes

Complete the table of values. For the inverse, switch the x and y.

Will the inverse be a function? no - does not pass VLT

Can you restrict the domain so that the inverse IS a function? How? yes

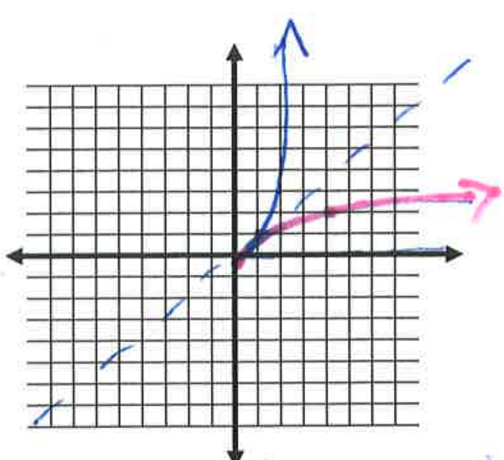
Now graph the *restricted* f(x) and its inverse, $f^{-1}(x)$.

For $f(x) = x^2$, **D:** [0, ∞)

R: [0, ∞)

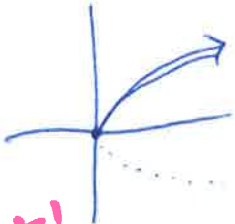
And for $f^{-1}(x) = \sqrt{x}$ **D:** [0, ∞)

R: [0, ∞)

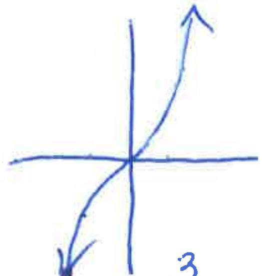


What observation can you make about the domains and ranges? they switch

1-1 IF D: [0, ∞) $y = x^2$

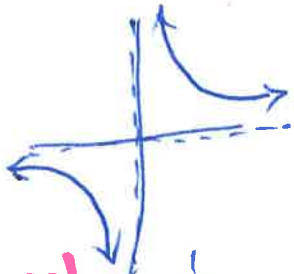


1-1 $y = \sqrt{x}$

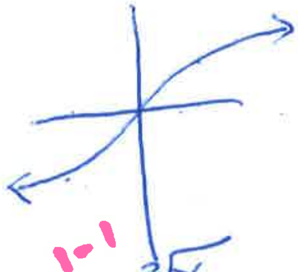


1-1 $y = x^3$

1-1 $y = x$



1-1 $y = \frac{1}{x}$

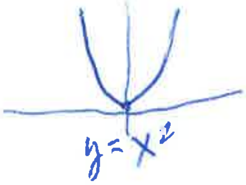


1-1 $y = \sqrt[3]{x}$

Inverse Functions

One-to-one function (or 1-1)

a function where every x has exactly one y
and " y " " " x



★ pass VLT AND HLT

How to check for one-to-one function?

Check these: $f(x) = x^2$ $f(x) = x^3$ $f(x) = x$ $f(x) = 1/x$ $f(x) = \sqrt{x}$ $f(x) = \sqrt[3]{x}$

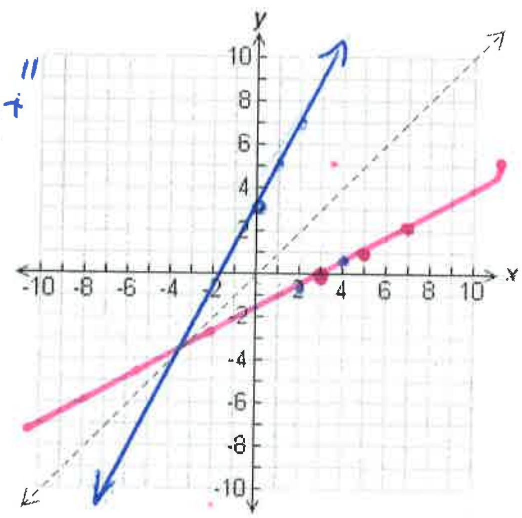
Which will have inverses that are functions? (see graphs above)

Which would if we restrict the domain of f(x)? (see graphs above)

EX. 1 Given $f(x) = 2x + 3$, complete the T-tables, switch the D and R values, and sketch both graphs on the same axes. Note the reflection over $y = x$

x	f(x)
-2	
-1	
0	
1	
2	

$f^{-1}(x)$ " f inverse of x"



x	$f^{-1}(x)$
	-2
	-1
	0
	1
	2

	f(x)	$f^{-1}(x)$
Domain (Restrict if needed so $f^{-1}(x)$ is a function.)	$(-\infty, \infty)$	$(-\infty, \infty)$
Range	$(-\infty, \infty)$	$(-\infty, \infty)$
End Behavior	$x \to \infty$ $y \to \infty$ $x \to -\infty$ $y \to -\infty$	$x \to \infty$ $y \to \infty$ $x \to -\infty$ $y \to -\infty$

II Finding the Inverse Algebraically

SWBAT: 1) Find the inverse of a function algebraically and

2) Give its domain and Range

Finding $f^{-1}(x)$ algebraically

1) subst. y for f(x)
2) switch x and y
3) solve for y=

Ex. $\cancel{f(x)} = 2x + 3$

$y = 2x + 3$

$x = 2y + 3$
$\quad -3 \qquad -3$

$\frac{x-3}{2} = \frac{\cancel{2}y}{\cancel{2}}$

$\boxed{f^{-1}(x) = \frac{x-3}{2}}$

or $\frac{1}{2}x - \frac{3}{2}$

In these examples, determine whether f(x) is one-to-one. If it is not, restrict the domain so that $f^{-1}(x)$ is one-to-one. Give the restricted domain and the range. Then find $f^{-1}(x)$ and write its domain and range.

Ex. $f(x) = \frac{3}{x+1}$ x^0

D: $(-\infty, -1) \cup (-1, \infty)$ VA

R: $(-\infty, 0) \cup (0, \infty)$ HA

$\cancel{f(x)}$ y $= \frac{3}{x+1}$

$x = \frac{3}{y+1}$

$\frac{\cancel{x}(y+1)}{\cancel{x}} = \frac{3}{x} - 1$

$y = \frac{3}{x} - 1$

$f^{-1}(x) = \frac{3}{x} - 1$

D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, -1) \cup (-1, \infty)$

Ex. $f(x) = \sqrt{x+3} - 2$

v: $(-3, -2)$

D: $[-3, \infty)$

R: $[-2, \infty)$

PEMDAS

$\cancel{f(x)}$ y $= \sqrt{x+3} - 2$

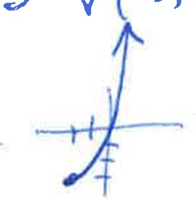
$x + 2 = \sqrt{y+3} \quad -2 \; +2$

$(x+2)^2 = (\sqrt{y+3})^2$

$(x+2)^2 - 3 = y + 3 - 3$

$y = (x+2)^2 - 3$

$f^{-1}(x) = (x+2)^2 - 3$ v $(-2, -3)$



D: $[-2, \infty)$

R: $[-3, \infty)$

Ex. $f(x) = 4\sqrt[3]{x-5}$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

y $\cancel{f(x)} = 4\sqrt[3]{x-5}$

$\left(\frac{x}{4}\right)^3 = \left(\cancel{4}\sqrt[3]{y-5}\right)^3$

$\left(\frac{x}{4}\right)^3 = y-5$

$+5 \qquad +5$

$y = \left(\frac{x}{4}\right)^3 + 5$

$f^{-1}(x) = \left(\frac{x}{4}\right)^3 + 5$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

V: $(-1,-7)$

Ex. $f(x) = 3(x+1)^2 - 7$

D: $[-1, \infty)$

R: $[-7, \infty)$

restricted to make the inverse a function

y $\cancel{f(x)} = 3(x+1)^2 - 7$

$x = 3(y+1)^2 - 7$

$+7 \qquad +7$

$\sqrt{\frac{x+7}{3}} = \sqrt{(y+1)^2}$

$= y + 1$ $\quad -1$

$y = \sqrt{\frac{x+7}{3}} - 1$

$(-7,-1)$

$f^{-1}(x) = \sqrt{\frac{x+7}{3}} - 1$

D: $[-7, \infty)$

R: $[-1, \infty)$

Ex. $f(x) = -(x-2)^3 + 1$

D:

R:

$f(x) = -(x-2)^3 + 1$

$f^{-1}(x) =$

D:

R: