Math 3 – Unit 1 Day 1 Name Key

Systems of Equations

A System of Equations A grouping of one or more equations containing one or more variables

Examples:

$$\begin{cases} x + y = 2 \\ 2x + y = 5 \end{cases} \qquad \begin{cases} 2y = x + 2 \\ y = 5x - 7 \\ 6x - y = 5 \end{cases}$$

Solutions to a System

To be a **solution to the system,** the values must make all equations in the system true

Is (-3, 4) a solution to the system?

$$\begin{cases} 3x + y = -5 \\ -x - y = -7 \end{cases}$$

Check: $3(-3) + (4) \stackrel{?}{=} -5$

$-9 + 4 = -5$

$-5 = -5$ ✓

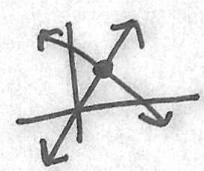
$-(-3) - (4) = -7$ $\boxed{\text{No}}$

$3 - 4 = -7$

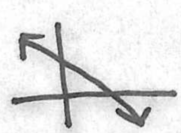
$-1 = -7$ ×

Types of Solutions

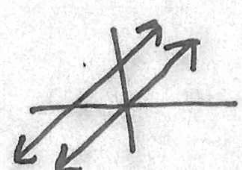
Intersection Lines have one unique solution.



Coincidental Lines (or same lines) have many solutions.



Parallel Lines have NO solutions.



Solving Systems of Equations

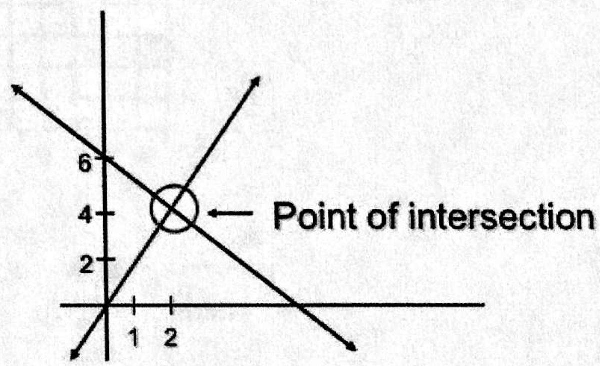
There are three methods to solving a system of equations.

1. Graphing
2. Substitution
3. Elimination

<u>Graphing</u>

How do we "solve" a system of equations? By finding the point where two or more equations intersect

$$\begin{cases} x + y = 6 \\ y = 2x \end{cases}$$



Standard Form of a Linear Equation: ax + by = c

The equation ax + by = c is just another form of a linear equation.

To graph, change it to y = mx + b OR use the x and y-intercepts

y-intercept

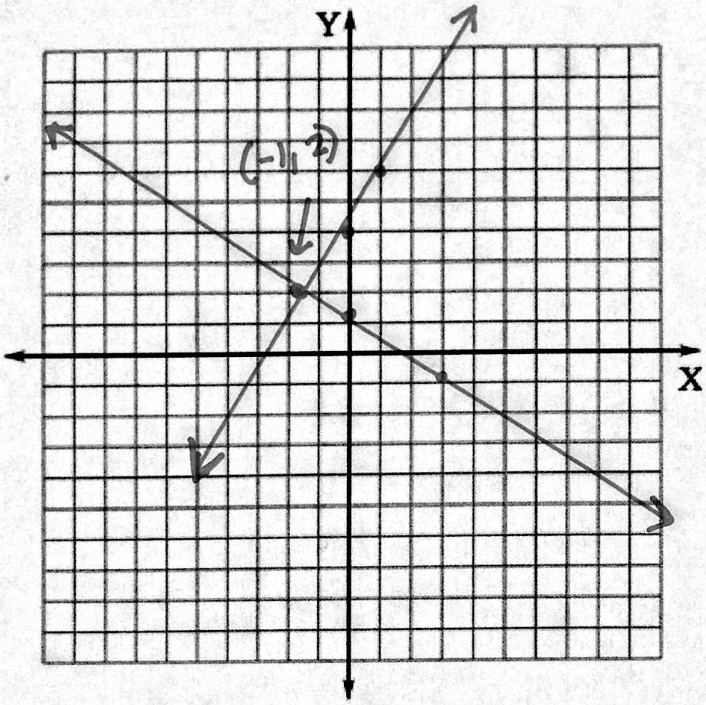
Ex) Change $2x + 3y = 6$ to standard from

$$3y = -2x + 6$$

$$y = -\frac{2}{3}x + \frac{6}{3}$$

$$y = -\frac{2}{3}x + 2$$

Ex) Solve the following system by Graphing Manually: $\begin{cases} 2x + 3y = 4 \\ y = 2x + 4 \end{cases}$



$2x + 3y = 4$

$3y = -2x + 4$

$y = -\frac{2}{3}x + \frac{4}{3}$

$y = 2x + 4$

solution:

$\boxed{(-1, 2)}$

Calculator Steps:

Step 1: Type both equations into $\boxed{Y=}$

Step 2: $\boxed{\text{2nd}}$ $\boxed{\text{Trace}}$

Step 3: $\boxed{5}$ Intersect

Step 4: $\boxed{\text{Enter}}$ $\boxed{\text{Enter}}$ $\boxed{\text{Enter}}$

Note: You may have to change your window to find the intersection!

Examples: Determine whether the following have one, none, or infinitely many solutions. If a solution exists, find the solution.

1.

$\begin{cases} 2y + x = 8 \\ y = 2x + 4 \end{cases}$

(0,4)

$\boxed{\text{one}}$

2.

$\begin{cases} y = -6x + 8 \\ y + 6x = 8 \end{cases}$

$\boxed{\text{infinitely many solutions}}$

(same line)

3.

$\begin{cases} x - 5y = 10 \\ -5y = -x + 6 \end{cases}$

parallel lines, so:

$\boxed{\text{none}}$

Solve by graphing:

1) $y = -\frac{5}{3}x + 3$

$y = \frac{1}{3}x - 3$

$\boxed{(3, -2)}$

Substitution If one of the equations is already solved for a variable, Substitution may be an easy method to solve.

1) Make sure one of the equations is in either y= form or x= form

2) Substitute the expression into the other equation.

3) Solve for the variable.

4) Substitute the value of x (or y) into the other equation and solve

Examples: Solve by Substitution.

1.

$$\begin{cases} 2x - 3y = 6 \\ y = -x - 12 \end{cases}$$

$2x - 3(-x - 12) = 6$

$2x + 3x + 36 = 6$

$5x + 36 = 6$

$5x = -30$

$x = -6$

$y = -x - 12$

$y = -(-6) - 12$

$y = 6 - 12$

$y = -6$

$\boxed{(-6, -6)}$

2.

$y = 4x + 3$

$y = -x - 2$

$y = 4x + 3$

$4x + 3 = -x - 2$

$5x = -5$

$x = -1$

$y = -(-1) - 2$

$y = 1 - 2$

$y = -1$

$\boxed{(-1, -1)}$

Elimination We can solve by elimination by either adding or subtracting two equations to eliminate a variable!

Example: Solve by Elimination

add

$$\begin{cases} 3x-2y=14 \\ +(2x+2y=6) \end{cases}$$

$5x = 20$

$x=4$

$\boxed{(4,-1)}$

$2(4)+2y=6$

$8+2y=6$

$2y=-2$

$y=-1$

subtract

$$\begin{cases} 4x+9y=1 \\ -(4x+6y=-2) \end{cases}$$

$3y=3$

$y=1$

$4x+9(1)=1$

$4x+9=1$

$4x=-8$

$x=-2$

$\boxed{(-2,1)}$

Note: If one will not cancel, multiply one or both equations to get variables to cancel

$7(\mathbf{5x+y=9})$

$\mathbf{10x-7y=-18}$

$\rightarrow$

$35x+7y=63$

$+(10x-7y=-18)$

$45x=45$

$x=1$

$10(1)-7y=-18$

$10-7y=-18$

$-7y=-28$

$y=4$

$\rightarrow$ $\boxed{(1,4)}$

Special Solutions Solve each system by elimination.

$$\begin{cases} 2x-y=3 \\ -2x+y=-3 \end{cases}$$

$0=0$

True statement

same line

$\boxed{\text{infinitely many solutions}}$

$$\begin{cases} 2x-3y=18 \\ -2x+3y=-6 \end{cases}$$

$0=12$ false

parallel lines

$\boxed{\text{no solution}}$