M3H - Solving Equations with Rational Exponents and Radicals

An "inverse operation" is an operation to undo another operation to isolate a variable. Why do we want to "isolate" a variable?

What is the inverse of:

taking a square root? squaring ²

a cube root? cubing ³

Raising to the 2/3 power? raising to the 3/2 power

To solve Radical Equations:

$(\sqrt[4]{a})^4 = (14)^4$ $a = 38416$	$(\sqrt[5]{b})^5 = (50)^5$ $b = 312{,}509000$
$\sqrt[5]{a^9} = 26$ $(a^{\frac{9}{5}})^{\frac{5}{9}} = (26)^{5/9}$ $a = 6.11$	$\sqrt[4]{b^3} = 27$ $(b^{3/4})^{4/3} = (27)^{4/3}$ $b = 81$
$(\sqrt[7]{a})^3 = 21$ $(a^{\frac{3}{7}})^{\frac{7}{3}} = (21)^{7/3}$ $a = 1216.69$	$(\sqrt{b})^5 = 12$ $(b^{\frac{1}{2}})^{\frac{2}{5}} = (12)^{\frac{2}{5}}$ $b = 2.7$

Solving Rational Exponent Equations

$(a^{\frac{1}{5}})^5 = (50)^5$ $a = 312{,}509000$	$(b^{\frac{1}{4}})^4 = (14)^4$ $b = 38{,}416$
$(a^{\frac{3}{4}})^{\frac{4}{3}} = (27)^{\frac{4}{3}}$ $a = 81$	$(b^{\frac{9}{5}})^{5/9} = (26)^{5/9}$ $b = 6.11$
$(a^{\frac{1}{2}})^5 = 12$ $(a^{\frac{5}{2}})^{\frac{2}{5}} = (12)^{2/5}$ $a = 2.7$	$(b^{\frac{1}{7}})^3 = 21$ $(b^{\frac{3}{7}})^{\frac{7}{3}} = (21)^{7/3}$ $b = 1216.69$

Solve the Following Equations:

$x^{1/4} - 2 = 3$

$(x^{\frac{1}{4}})^4 = (5)^4$

$\boxed{x = 625}$

$4x^7 - 6 = -2$

$4x^7 = 4$

$x^7 = 1$

$\sqrt[7]{x^7} = \sqrt[7]{1} \rightarrow \boxed{x = 1}$

$3(x^{2/3} + 5) = 207$

$x^{\frac{2}{3}} + 5 = 69$

$(x^{\frac{2}{3}})^{\frac{3}{2}} = (64)^{\frac{3}{2}}$

$\boxed{x = 512}$

$1450 = 800\left(1 + \frac{x}{12}\right)^{7.8}$

$\frac{29}{16} = \left(1 + \frac{x}{12}\right)^{\frac{39}{5}}$

$\left(\frac{29}{16}\right)^{\frac{5}{39}} = \left(\left(1 + \frac{x}{12}\right)^{\frac{39}{5}}\right)^{\frac{5}{39}}$

$1.079 = 1 + \frac{x}{12} \rightarrow \boxed{x = .95}$

$3\sqrt[3]{x - 2} = 12$

$\sqrt[3]{x-2} = 4$

$(\sqrt[3]{x-2})^3 = (4)^3$

$x - 2 = 64$

$\boxed{x = 66}$

$\sqrt{a + 2} - 2 = 12$

$\sqrt{a+2} = 14$

$a + 2 = 196$

$\boxed{a = 194}$

$2 - 3\sqrt{2x - 5} = -33$

$-3\sqrt{2x-5} = -35$

$\sqrt{2x-5} = \frac{35}{3}$

$2x - 5 = \frac{1225}{9}$

$\boxed{x = \frac{635}{9}}$

$2\sqrt[4]{3x + 1} - 10 = 0$

$2\sqrt[4]{3x+1} = 10$

$\sqrt[4]{3x+1} = 5 \rightarrow (\sqrt[4]{3x+1})^4 = (5)^4$

$3x + 1 = 625$

$3x = 624$

$\boxed{x = 208}$

Some of these problems have **NO SOLUTION.**

Some have **EXTRANEOUS SOLUTIONS.**

Extraneous solutions are: solutions that make the statement false.

SO...ALWAYS CHECK YOUR ANSWERS!

Solve the following equations:

$\sqrt{a+2} - 2 = a$

$\sqrt{a+2} = a+2$

$(\sqrt{a+2})^2 = (a+2)^2$

$a+2 = a^2+4a+4$

$0 = a^2+3a+2$

$0 = (a+2)(a+1)$

$a = -2$ $\quad a = -1$

Check:

$\sqrt{-2+2} - 2 \stackrel{?}{=} -2$

$\sqrt{0} - 2 \stackrel{?}{=} -2$

$-2 = -2$ ✓

$\sqrt{-1+2} - 2 = -1$

$\sqrt{1} - 2 = -1$

$1 - 2 = -1$

$-1 = -1$ ✓

$\sqrt{3x-2} = -5$

$3x - 2 = 25$

$3x = 27$

$\boxed{x = 9}$

$(2x+7)^{1/2} - x = 2$

$(2x+7)^{\frac{1}{2}} = x+2$

$\left((2x+7)^{\frac{1}{2}}\right)^2 = (x+2)^2$

$2x+7 = x^2+4x+4$

$0 = x^2+2x-3$

$0 = (x+3)(x-1)$

$\boxed{x = -3}$ $\quad \boxed{x = 1}$

Check:

$\boxed{-3}$ $(2(-3)+7)^{\frac{1}{2}} - -3 \stackrel{?}{=} 2$

$4 = 2$ X

$\boxed{1}$: $(2(1)+7)^{\frac{1}{2}} - 1 \stackrel{?}{=} 2$

$3 - 1 \stackrel{?}{=} 2$

$2 = 2$ ✓

$3x^{4/3} + 5 = 53$

$3x^{\frac{4}{3}} = 48$

$x^{\frac{4}{3}} = 16$

$\left(x^{\frac{4}{3}}\right)^{\frac{3}{4}} = (16)^{\frac{3}{4}}$

$\boxed{x = 8}$

Check:

$3(8)^{4/3} + 5 \stackrel{?}{=} 53$

$53 = 53$ ✓