Key

U4 Day 10 Homework Geometric Sequences and Series

Given the explicit formula for a geometric sequence, find the first five terms and the 8th term.

1. $a_n = 3^{n-1}$ 1, 3, 9, 27, 81 $a_8 = 2187$

2. $a_n = 2 \cdot (\frac{1}{4})^{n-1}$ 2, $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{32}$, $\frac{1}{128}$ $a_8 = .000122$

3. $a_n = -4 \cdot 3^{n-1}$ -4, -12, -36, -108, -324 $a_8 = -8748$

Given the recursive formula for a geometric sequence, find the common ratio, the first five terms and the explicit formula $a_n = a_1 \cdot r^{n-1}$

1. $a_n = a_{n-1} \cdot 2$
   $a_1 = 2$
   $r = 2$
   2, 4, 8, 16, 32
   $a_n = 2(2)^{n-1}$

2. $a_n = a_{n-1} \cdot -3$
   $a_1 = -3$
   $r = -3$
   -3, 9, -27, 81, -243
   $a_n = -3(-3)^{n-1}$

Find the partial sum S_n of the geometric sequence that satisfies the given conditions. $S_n = \frac{a_1(1-r^n)}{1-r}$

1. $a_1 = 5, r = 2, n = 6$ $S = \frac{5(1-2^6)}{1-2} = \boxed{315}$

2. $a_1 = \frac{2}{3}, r = \frac{1}{3}, n = 4$ $S = \frac{\frac{2}{3}(1-(\frac{1}{3})^4)}{1-\frac{1}{3}} = \boxed{\frac{80}{81}}$

3. $a_1 = 28, r = -2, n = 6$ $S = \frac{28(1-(-2)^6)}{1--2} = \boxed{-588}$

4. $a_1 = 0.12, r = -3, n = 4$ $S = \frac{0.12(1-(-3)^4)}{1-(-3)} = \boxed{-\frac{12}{5}}$

Determine whether each infinite geometric series converges. If it converges, find the sum.

5. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$ $\times\frac{1}{3}$ $r = \frac{1}{3} \rightarrow$ converges to $S = \frac{1}{1-\frac{1}{3}} = \boxed{\frac{3}{2}}$

6. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$ $\times -\frac{1}{2}$ $r = -\frac{1}{2} \rightarrow |-\frac{1}{2}| < 1 \rightarrow$ converges to $S = \frac{1}{1--\frac{1}{2}} = \boxed{\frac{2}{3}}$

7. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$ $\times -\frac{1}{3}$ $|-\frac{1}{3}| < 1 \rightarrow$ converges to $S = \frac{1}{1--\frac{1}{3}} = \boxed{\frac{3}{4}}$

8. $\frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \cdots$ $\frac{2}{5}$ $|\frac{2}{5}| < 1 \rightarrow$ converges to

$S = \frac{\frac{2}{5}}{1-\frac{2}{5}} = \boxed{\frac{2}{3}}$