U4 Day 9 Geometric Sequence Notes

A geometric sequence is a sequence of numbers with a common ratio between terms. For example, 3, 6, 12, 24... is a geometric sequence with the first term 3 and a common ratio of 2. We will typically denote the first term of a geometric sequence as a_1 and its common ratio as r.

$a_n = a_1 r^{n-1}$, where a_n denotes the n[th] term of the sequence. Fill in the information below.

Ex. 3, 6, 12, 24, 48, 96, 192 $a_1 =$ 3, $r =$ 2 $a_n =$ $3(2)^{n-1}$ $a_{10} =$ $3 \cdot 2^9 = 1536$

Recursive formula $a_n =$ $a_{n-1}(2)$

Individual Practice: Give the next 3 terms in the sequence, then list the a_1 and d values for each example below. Then write an equation describing the pattern.

1. 10, 30, 90, 270, 810, 2430 $a_1 =$ 10, $r =$ 3 $a_n =$ $10(3)^{n-1}$ $a_{11} =$ $10 \cdot 3^{10} = 590{,}490$

   Recursive formula $a_n =$ $a_{n-1}(3)$

2. 16, 8, 4, 2, 1, ½ $a_1 =$ 16, $r =$ ½ $a_n =$ $16(\frac{1}{2})^{n-1}$ $a_{12} =$ $16(\frac{1}{2})^{11} = \frac{1}{128}$

   ★ use fractions

   Recursive formula $a_n =$ $a_{n-1}(\frac{1}{2})$

3. −2, −10, −50, −250, −1250 $a_1 =$ −2, $r =$ 5 $a_n =$ $-2(5)^{n-1}$ $a_9 =$ $-2(5)^8 = -781250$

   Recursive formula $a_n =$ $a_{n-1}(5)$

4. −1, 7, −49, 343, −2401 $a_1 =$ −1, $r =$ −7 $a_n =$ $-1(-7)^{n-1}$ $a_8 =$ $-1(-7)^7 = 823543$

   Recursive formula $a_n =$ $a_{n-1}(-7)$

Sequences with "n"

Sequences with "n" where the formula is given
n = number of term in the sequence

ex. 2, 4, 6, 8, ... 2 is n=1, 4 is n=2, 6 is n=3, 8 is n=4...

Find the first 4 terms of the following: EXPLICIT FORM (Plug in 1, 2, 3)

1) a_n=7n 7, 14, 21, 28

2) a_n=n²+5 6, 9, 14, 21

3) $a_n = \frac{1}{4}n + 2$ $\frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3$

U4 Day 9 Geometric Series Notes

A geometric series is the sum of a geometric sequence. There are finite (partial) and infinite geometric series.

Finite Geometric Series

Sequence: 3, 6, 12, 24, ... Series: 3 + 6 + 12 + 24 + ⋯ $a_1 = 3$ $r = 2$

$S_n = \frac{a_1(1-r^n)}{1-r}$, where n is the number of terms, and a_1 is the 1[st] term.

Ex. Find the sum of the first 10 terms. Series: 3 + 6 + 12 + 24 + ⋯

Use the formula to find S_{10} $S_{10} = \frac{3(1-2^{10})}{1-2} = 3069$

Individual Practice: Find the sum indicated.

Same as 1-4

5. 10, 30, 90, 270, 810, 2430 $a_1 =$ 10, $r =$ 3, $S_{15} = \frac{10(1-3^{15})}{1-3} = 71744530$
6. 16, 8, 4, ____, ____, ____ $a_1 =$ 16, $r =$ ½, $S_9 = \frac{16(1-\frac{1}{2}^9)}{1-\frac{1}{2}} = 31.9375$
7. −2, −10, −50, ____, ____ $a_1 =$ −2, $r =$ 5, $S_{12} = \frac{-2(1-5^{12})}{1-5} = -122070312$
8. −1, 7, −49, ____, ____ $a_1 =$ −1, $r =$ −7, $S_{10} = \frac{-1(1-(-7)^{10})}{1+7} = 35309406$

Infinite Geometric Series

You can only find the sum of an infinite geometric series if the series converges (approaches a distinct number).

If $|r| < 1$, the series is "convergent" If $|r| > 1$, the series is "divergent"

$S = \frac{a_1}{1-r}$, where a_1 is the 1[st] term, and r is the common ratio, ONLY if the series diverges.

Ex. Find the sum, if possible. Series: $-2 - \frac{2}{3} - \frac{2}{9} - \cdots$

1. First determine if it converges. Is $|r| < 1$? $r = \frac{1}{3}$ converges ✓
2. If it does converge, use the formula. $S = \frac{a_1}{1-r} = \frac{-2}{1-\frac{1}{3}} = -3$

Individual Practice: Find the sum, if possible. (If it is not convergent, then just put "can't find" next to $S =$)

1. $16 + 12 + 9 + \cdots$ $r =$ .75 convergent? yes $S = \frac{16}{1-.75} = 64$
2. $5 + 7.5 + 11.25 + \cdots$ $r =$ 1.5 convergent? no $S =$ not possible
3. $10 + 5 + 2.5 + \cdots$ $r =$ ½ convergent? yes $S = \frac{10}{1-\frac{1}{2}} = 20$
4. $-\frac{2}{3} + \frac{1}{9} - \frac{1}{54} + \cdots$ $r =$ $-\frac{1}{6}$ convergent? yes $S = \frac{-\frac{2}{3}}{1+\frac{1}{6}} = -\frac{4}{7}$