Key

M3H U4 Day 12 HW Sequence and Series Applications

1. The first year a toy manufacturer introduces a new toy, its sales total $495,000. The company expects its sales to drop 10% each succeeding year. Find the total expected sales in the first 6 years. Find the total expected sales if the company offers the toy for sale for as long as anyone buys it.

$a_n = 495{,}000(.9)^{n-1}$

Total for first six years: $\frac{495{,}000(1-.9^6)}{1-.9} = \$2{,}319{,}367.05$

$|.9| < 1$ so converges:

$S_\infty = \frac{495{,}000}{1-.9}$

$S_\infty = 4{,}950{,}000$

2. When a pendulum swings freely, the length of its arc decreases geometrically. Find each missing arc length.

a. 20th arc is 20 in.; 22nd arc is 18.5 in.

$\underset{n=20}{20}, \boxed{\underset{n=21}{19.24}}, \underset{n=22}{18.5}$ $\quad r^2 = \frac{18.5}{20}$

$r = .962$

b. 8th arc is 27 mm; 10th arc is 3 mm

$\underset{n=8}{27}, \boxed{\underset{n=9}{9}}, \underset{n=10}{3}$ $\quad r^2 = \frac{3}{27}$

$r = \frac{1}{3}$

3. A bookshelf has 7 shelves of different widths. Each shelf is narrower than the shelf below it. The bottom three shelves are 36 in., 31 in., and 26 in. wide. Arithmetic. $a_n = 36 + -5(n-1)$

$a_n = 41 - 5n$

a. The shelf widths decrease by the same amount from bottom to top. What is the width of the top shelf?

$n = 7$

$a_7 = 41 - 5(7) = \boxed{6 \text{ inches}}$

b. What is the total shelf space of all seven shelves?

$n = 7$, $a_1 = 36$, $a_n = 6$ $\rightarrow$ $S_7 = \frac{7}{2}(36+6) = \boxed{147 \text{ in}}$

4. A man swims 1.5 mi on Monday, 1.6 mi on Tuesday, 1.8 mi on Wednesday, 2.1 mi on Thursday, and 2.5 mi on Friday. If the pattern continues, how many miles will he swim on Saturday?

$\underset{M}{1.5}, \underset{T}{1.6}$ (+.1), $\underset{W}{1.8}$ (+.2), $\underset{Th}{2.1}$ (+.3), $\underset{F}{2.5}$ (+.4), $\underset{S}{3}$ (+.5) ← 3 miles on Saturday

5. A teacher donates the same amount of money each year to help protect the rainforest. At the end of the second year, she has donated enough money to protect 8 acres. At the end of the third year, she has donated enough money to protect 12 acres. How many acres will the teacher's donations protect at the end of the tenth year?

$\underset{n=1}{4}, \underset{n=2}{8}, \underset{n=3}{12}, \ldots, \underset{n=10}{?}$

-4, +4 acres

$a_n = a_1 + d(n-1)$

$a_{10} = 4 + 4(10-1)$

$= 4 + 36 = \boxed{40 \text{ acres}}$

6. A seashell has chambers that are each 0.82 times the length of the enclosing chamber. The outer chamber is 32 mm around. Find the total length of the shell's spiraled chambers. ~~7. An embroidery pattern calls for five stitches in the first row and for three more stitches in each successive row. The 25th row, which is the last row, has 77 stitches. Find the total number of stitches in the pattern.~~

$32, 26.24, 21.52, \ldots$

$S_\infty = \frac{32}{1-.82} = \boxed{177.78 \text{ mm}}$

7. An embroidery pattern calls for five stitches in the first row and for three more stitches in each successive row. The 25th row, which is the last row, has 77 stitches. Find the total number of stitches in the pattern.

$\underset{n=1}{5}, \underset{n=2}{8}, \underset{n=3}{11}, \ldots, \underset{n=25}{77}$ $\quad S_{25} = \frac{25}{2}(5+77) = \boxed{1025}$

8. The first figure of a fractal contains one segment. For each successive figure, six segments replace each segment.

a. How many segments are in each of the first four figures of the sequence? b. Write a recursive definition for the sequence.

$\underset{n=1}{1} + \underset{n=2}{6} + \underset{n=3}{36} + \underset{n=4}{216} =$ **259 segments**

$a_n = a_{n-1} \cdot 6$, $a_1 = 1$

9. The deer population in an area is increasing. This year, the population was 1.025 times last year's population of 2537. A). Assuming that the population increases at the same rate for the next few years, write an explicit formula for the sequence. B). Find the expected deer population for the fourth year of the sequence.

A) $a_n = 2537(1.025)^{n-1}$

B) $a_4 = 2537(1.025)^{4-1} =$ **2732 deer**

10. This month, your friend deposits $400 to save for a vacation. She plans to deposit 10% more each successive month for the next 11 months. How much will she have saved after the 12 deposits?

$\underset{n=1}{400}, \underset{n=2}{440}, \underset{n=3}{484}, \ldots, \underset{n=12}{}$

$a_n = 400(1.10)^{n-1}$

$S_{12} = \frac{400(1-1.1^{12})}{1-1.1} =$ **$8,553.71**

11. Each year, a volunteer organization expects to add 5 more people to the number of shut-ins for whom the group provides home maintenance services. This year, the organization provides the service for 32 people.

a. Write a recursive formula for the number of people the organization expects to serve each year.

$a_1 = 32$ $\quad a_n = a_{n-1} + 5$

b. Write the first five terms of the sequence.

32, 37, 42, 47, 52

c. Write an explicit formula for the number of people the organization expects to serve each year.

$a_n = 32 + 5(n-1) \rightarrow a_n = 27 + 5n$

d. How many people would the organization expect to serve in the 20th year?

$a_{20} = 27 + 5(20) =$ **127 people**

12. To train new employees, an employer offers a bonus after 30 work days as follows. An employee must turn in one report on the first day; the number of reports for each subsequent day must increase by two. What is the minimum number of reports an employee will have to turn in over the 30 days to earn the bonus?

$\underset{n=1}{1}, \underset{n=2}{3}, \underset{n=3}{5}, \ldots \underset{n=30}{59}$

$a_{30} = 1 + 2(30-1)$

$a_{30} = 59$

$S_{30} = \frac{30}{2}(1+59) =$ **900 reports**