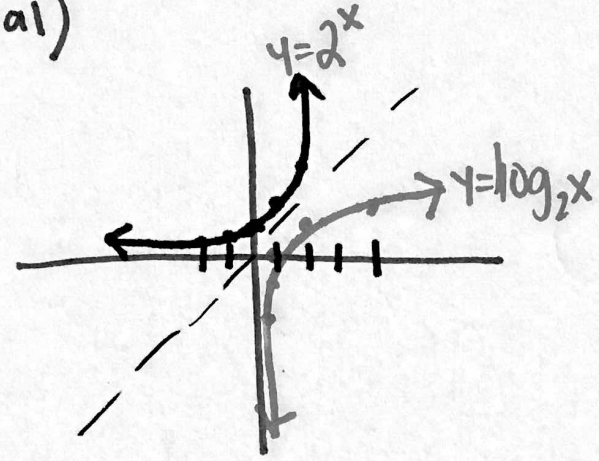
Log Functions as Inverses (unit 5 Day 3)

* The inverse of an exponential function is a logarithmic function

$y = 2^x$ (Exponential)

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4



$y = \log_2 x$ "what power can I raise 2 to to get 1?" "2 to what power gives you 1?"

x	y
1	0
2	1
4	2
1/2	-1
1/4	-2

↑ x

(ex) $\log_2 8 = 3 \longleftrightarrow 2^3 = 8$

$\log_3 9 = 2 \longleftarrow 3^2 = 9$

$y = \log_b x \leftrightarrow y = b^x$

* Logarithmic → Exponential

① $\log_3 8 = x$ → $3^x = 8$

② $\log_5 25 = 2$ → $5^2 = 25$

③ $\log_6 x = 36$ → $6^{36} = x$

* Evaluate the Log

(ex) $\log_{13} 169 = x$ → 2 — "13 to what power gives you 169?"

(ex) $\log_2 1024 = 10$

(ex) $\log_3 81 = 4$

(ex) $\log_5 1 = 0$

(ex) $\log_{25} 5 = \frac{1}{2}$

* Common Logarithms (The base is 10 but it's not written)

(ex) $\log_{10} x = 2$ — "10 raised to the 2nd power equals what?"

$10^2 = x$

$100 = x$

(ex) $\log_{10} 100 = 2$ ← "10 to what power is 100?"

(ex) $\log_{10} 1000 = 3$

(ex) $\log_{10} 10 = 1$

(ex) $\log_{10} 1 = 0$

(ex) $\log_{10} x = 4$

$10^4 = x$

$10{,}000 = x$

* Finding the inverse functions

(ex) $f(x) = \log_4 x^5$

$y = \log_4 x^5$

$x = \log_4 y^5$

$4^x = y^5$

$(4^x)^{\frac{1}{5}} = (y^5)^{\frac{1}{5}}$

$4^{\frac{x}{5}} = y$

$4^{\frac{x}{5}} = f^{-1}(x)$ or $\sqrt[5]{4^x} = f^{-1}(x)$

(ex) $y = \log_3(4x-4)$

$x = \log_3(4y-4)$

$3^x = 4y-4$

$3^x + 4 = 4y$

$\frac{3^x+4}{4} = y$

(ex) $y = 6\log_2(2x-7)$

$x = 6\log_2(2y-7)$

$\frac{x}{6} = \log_2(2y-7)$

$2^{\frac{x}{6}} = 2y-7$

$\frac{2^{\frac{x}{6}}+7}{2} = f^{-1}(x)$