Properties of Logs (Units Day 4)

* Two special bases
  - → common logs: Base 10
  - → Natural Log (ln): Base e ($\ln \rightarrow \log_e$)

ex) $\log_{10} 10 = 1$ this means $10^1 = 10$

$\log 1 = 0$ means $10^0 = 1$

$\log 49 = 1.69$ means $10^{1.69} \approx 49$

$\log 4 = .602$ means $10^{.602} \approx 4$

ex) $\ln e \Rightarrow \log_e e = (1)$ so this means $e^1 = e$

$\ln 1 \Rightarrow \log_e 1 = (0)$

$\ln 49 \approx 3.892$

$\ln 4 \approx 1.386$ so this means $e^{1.386} \approx 4$

* Change of Base Formula: $\log_x y = \dfrac{\log y}{\log x}$ ← common logs

ex) $\log_5 29 \Rightarrow \dfrac{\log 29}{\log 5} \Rightarrow 2.092$

ex) $\log_7 25 \approx \boxed{1.65}$

↳ $\dfrac{\log 25}{\log 7}$

* Properties of any logarithm

single → expanded

* **Product Rule**: $\log_b MN = \log_b M + \log_b N$

* **Quotient Rule**: $\log_b \frac{M}{N} = \log_b M - \log_b N$

* **Power Rule**: $\log_b M^n = n \log_b M$

* Expanding Logarithms

(ex) $\log_5 x^2 y^3$

$\log_5 x^2 + \log_5 y^3$

$2\log_5 x + 3\log_5 y$

(ex) $\ln \frac{2rs}{5w}$

$\ln 2rs - \ln 5w$

$(\ln 2 + \ln r + \ln s) - (\ln 5 + \ln w)$

$\ln 2 + \ln r + \ln s - \ln 5 - \ln w$

(ex) $\ln (3xyz)^2$

$2 \ln (3xyz)$

$2(\ln 3 + \ln x + \ln y + \ln z)$

$2\ln 3 + 2\ln x + 2\ln y + 2\ln z$

* Condensing Logarithms

(ex) $2\log x + 3\log 3y - 4\log w$

$\log \frac{x^2 (3y)^3}{w^4} = \log \frac{x^2 9y^3}{w^4}$

$= \log \frac{x^2 27y^3}{w^4}$

(ex) $\frac{1}{2}(\ln x - \ln y)$

$= \frac{1}{2}\left(\ln \frac{x}{y}\right)$

$= \ln \left(\frac{x}{y}\right)^{\frac{1}{2}}$

$= \ln \left(\frac{x}{y}\right)^{\frac{1}{2}}$

$= \ln \frac{\sqrt{x}}{\sqrt{y}}$

(ex) $\ln 7 - \ln 3 + \ln 6$

$\ln \frac{7}{3} + \ln 6$

$\ln \left(\frac{7}{3}\right)(6)$

$\ln 14$