Name: KEY Period: ____ Date: _____

REVIEW: Exponential and Logarithmic Expressions

Show all work on a separate sheet of paper! NO CALCULATORS!!!

Rewrite the logarithmic equations in exponential form.

1. $\log_5 125 = 3$ $5^3 = 125$	2. $\log_8 2 = \frac{1}{3}$ $8^{1/3} = 2$	3. $\log_3 81 = 4$ $3^4 = 81$	4. $\log_{50} 1 = 0$ $50^0 = 1$	5. $\log_3 \frac{1}{27} = -3$ $3^{-3} = \frac{1}{27}$

Simplify the expressions.

6. $(3^{1/3} \cdot 4^{1/3})^3$ 12	7. $\sqrt[3]{-8x^3y^4z^3}$ $-2xyz\sqrt[3]{y}$	8. $e^2(2e^4)^3$ $8e^{14}$	9. $7^{\log_7 x}$ x
10. $\left(\frac{4xy^{-1}}{16xy^2}\right)$ $\frac{1}{4y^3}$	11. $\left(\frac{1}{3}e^{-2}\right)^{-4}$ $81e^8$	12. $\left(\frac{e^{3x}}{2e}\right)^2$ $\frac{e^{6x}}{4e^2}$	13. $(3e)^2$ $9e^2$
14. $\log_3 27^x$ $x \log_3 27 = 3x$	15. $\sqrt[3]{8e^{12x}}$ $2e^{4x}$	16. $\frac{4e^4}{e^5} \cdot \frac{e}{-2}$ -2	17. $\sqrt[4]{16x^3y^4z^8}$ $2yz^2\sqrt[4]{x^3}$

Evaluate the expressions.

18. $81^{1/2}$ 9	19. $\log_2 64$ 6	20. $\log_3 \sqrt{3}$ $1/2$	21. $\log \frac{1}{1000}$ -3
22. $(-27)^{2/3}$ 9	23. $\log_2 \frac{1}{4}$ -2	24. $\ln e^2$ 2	25. $\log 100{,}000$ 5
26. $\log_{1/2} 8$ -3	27. $\log_2 1$ 0	28. $\log_{27} \frac{1}{9}$ $-2/3$	29. $\log_{100} \frac{1}{1000}$ -2

Expand the expressions.

30. $\log(8x)$ $\log 8 + \log x$	31. $\log_4 \frac{2xy^{-1}}{z}$ $\log_4 2 + \log_4 x + -1\log_4 y - \log_4 z$	32. $\log\sqrt{3xy}$ $\frac{1}{3}(\log 3 + \log x + \log y)$
33. $\log_2 \frac{(xy)^4}{z^2}$ $4(\log_2 x + \log_2 y) - 2\log_2 z$	34. $\ln \frac{3y}{\sqrt[4]{x}}$ $\ln 3 + \ln y - \frac{1}{4}\ln x$	35. $\log(3xyz^2)^3$ $3(\log 3 + \log x + \log y + 2\log z)$

Condense the expressions.

36. $2\log_3 7 - \log_3 x$ $\log_3\left(\frac{49}{x}\right)$	37. $\log 3 - \log 4 - \log 7$ $\log\left(\frac{3}{28}\right)$	38. $\ln x - \ln y + \ln z + 2\ln 3$ $\ln\left(\frac{x}{y} \cdot z \cdot 9\right)$
39. $\frac{1}{2}\log_2(x+5) - 2\log_2 x + \log_2 y$ $\log_2\left(\frac{\sqrt{x+5}}{x^2 y}\right)$	40. $\frac{1}{2}\log x - \log 4$ $\log\left(\frac{\sqrt{x}}{4}\right)$	41. $\log_3 4 + 2(\log_3 x - \log_3 5)$ $\log_3\left(4 \cdot \frac{x^2}{5^2}\right)$

Find the inverse, $f^{-1}(x)$, of each function.

42. $f(x) = (x-1)^2$ $\pm\sqrt{x} = y^{-1}$	43. $f(x) = 2x^3 + 3$ $\sqrt[3]{\frac{x-3}{2}} = y^{-1}$	44. $f(x) = \sqrt{x} - 2$ $(x+2)^2 = y^{-1}$
45. $f(x) = \ln(x-2) + 1$ $e^{x-1} + 2 = y^{-1}$	46. $f(x) = \log_2(7x)$ $\frac{2^x}{7} = y^{-1}$	47. $f(x) = \log(3x+2)$ $\frac{10^x - 2}{3} = y^{-1}$
48. $f(x) = 5^x + 2$ $\log_5(x-2) = y^{-1}$	49. $f(x) = \left(\frac{1}{3}\right)^x$ $\log_{\frac{1}{3}}(x) = y^{-1}$	50. $f(x) = -4^{x+2} - 5$ $\log_4(-x-5) - 2 = y^{-1}$

key Date: _____

Arithmetic and Geometric Sequences
Worksheet

Arithmetic Sequence - is a sequence of terms that have a common difference between them.

General Term:

$a_n = a_1 + d(n-1)$

Geometric Sequence - is a sequence of terms that have a common ratio between them.

General Term:

$a_n = a_1(r)^{n-1}$

1. Are the following sequences arithmetic, geometric, or neither? If they are arithmetic, state the value of d. If they are geometric, state r.

a) 6, 12, 18, 24, ... arithmetic $d=6$

b) 6, 11, 17, ... neither

c) 2, 14, 98, 686, ... geometric $r=7$

d) 160, 80, 40, 20, ... geometric $r=\frac{1}{2}$

e) -40, -25, -10, 5, ... arithmetic $d=15$

f) 7, -21, 63, -189, ... geometric $r=-3$

2. For the following arithmetic sequences, find a and d and state the formula for the general term. (explicit)
Don't forget to simplify!

a) - 10, - 4, 2, 8, 14, ...

$a_1=-10$ $d=6$

$a_n = -10+6(n-1)$

$a_n = -10+6n-6$

$a_n = -16+6n$

b) 10, 8, 6, 4, ...

$a_1=10$ $d=-2$

$a_n = 10-2(n-1)$

$a_n = 10-2n+2$

$a_n = 12-2n$

c) 36, 31, 25, 21, ...

$a_1=36$ $d=-5$

$a_n = 36-5(n-1)$

$a_n = 36-5n+5$

$a_n = 41-5n$

3. Use your formula from ~~question 2c~~ to find the values of t_7 and t_{20}.

$a_7 = 6$

$a_{20} = -59$

explicit!

4. For the following geometric sequences, find a and r and state the formula for the general term.

a) 1, 3, 9, 27, ...

$a_1 = 1 \quad r = 3$

$a_n = 1(3)^{n-1}$

b) 12, 6, 3, 1.5, ...

$a_1 = 12 \quad r = \frac{1}{2}$

$a_n = 12\left(\frac{1}{2}\right)^{n-1}$

c) 9, -3, 1, ...

$a_1 = 9 \quad r = \frac{-1}{3}$

$a_n = 9\left(\frac{-1}{3}\right)^{n-1}$

5. Use your formula from question 4c) to find the values of the t_4 and t_{12}

$a_4 = \frac{-1}{3}$

$a_{12} = -5.08 \times 10^{-5}$

6. Find the number of terms in the following arithmetic sequences. Hint: you will need to find the formula for t_n first! a_n

a) 2, 5, 8, ... , 299

$a_n = 2 + 3(n-1)$

$a_n = 2 + 3n - 3$

$a_n = -1 + 3n$

$299 = -1 + 3n \rightarrow n = 100$

b) 9, 5, 1, ... - 251.

$a_n = 9 + -4(n-1)$

$a_n = 9 - 4n + 4$

$a_n = 13 - 4n$

$-251 = 13 - 4n$

$66 = n$

Answers:
1a) arithmetic d = 6 b) neither c) geometric r = 7 d) geometric r = 0.5 or r = ½ e) arithmetic d = 15 f) geometric r = -3 2a) a = -10; d=6; t_n=6n-16 b) a = 10; d=-2; t_n=-2n+12 c) a = 36; d=-5; t_n= - 5n+41 3. t_7=6; t_{20} = -59 4. a) a = 1; r = 3; $t_n = 1(3)^{n-1}$ b) a = 12; r = $\frac{1}{2}$; $t_{12} = 12\left(\frac{1}{2}\right)^{n-1}$ c) a = 9; r = - 3; $t_n = 9(-3)^{n-1}$ d) t_4 = -243 t_{12} = -177147 6. a) t_n = 3n-1; n = 100 b) t_n = -4n+13; n=66

Math 3H Unit 5 Test 2 Review Key

① $y = \log_2(x+5) - 9$

$y + 9 = \log_2(x+5)$

$2^{y+9} = x + 5$

$2^{y+9} - 5 = x$

$\boxed{f^{-1}(x) = 2^{x+9} - 5}$

$y = -7\log_6(-3x)$

$y/-7 = \log_6(-3x)$

$6^{y/-7} = -3x$

$(6^{y/-7})/-3 = x$

$\boxed{f^{-1}(x) = \frac{6^{-x/7}}{-3}}$ ← NO, you cannot divide 6 by −3!

② $\log(16+2b) = \log(b^2-4b)$ ← Quadratic!

$16 + 2b = b^2 - 4b$

$0 = b^2 - 6b - 16$

$0 = (b-8)(b+2)$

✓ $\boxed{b = 8}$ $\boxed{b = -2}$ ✓

$\log_9(x+6) - \log_9 x = \log_9 2$

$\log_9\left(\frac{x+6}{x}\right) = \log_9 2$

$\frac{x+6}{x} = 2$

$x + 6 = 2x$

$\boxed{6 = x}$ ✓

$7 - 2^{x+7} = 5$

$-2^{x+7} = -2$ ← Divide by −1!

$2^{x+7} = 2$

$x + 7 = 1$

$\boxed{x = -6}$ ✓

$11^{n-8} - 5 = 54$

$11^{n-8} = 59$

$\log_{11}(59) = n - 8$

$\boxed{9.70 = n}$

③ $\log(12) + \frac{1}{2}\log(7) - \log(2)$

$\log 12\sqrt{7} - \log 2$

$\log \frac{12\sqrt{7}}{2} = \boxed{\log 6\sqrt{7}}$

$3\ln(a) - 2\ln(b) - 4\ln(c)$

$\ln a^3/b^2 - \ln c^4$

$\boxed{\ln \frac{a^3}{b^2c^4}}$

④ $\log_5\sqrt{\frac{x-1}{x+1}} = \log_5\sqrt{x-1} - \log_5\sqrt{x+1}$

$\boxed{\frac{1}{2}\log_5(x-1) - \frac{1}{2}\log_5(x+1)}$

$\log_5\left(\frac{x}{2}\right)$

$\boxed{\log_5(x) - \log_5(2)}$

⑤ $a_1 = 3$ $\boxed{r = -5}$

$\boxed{a_n = 3(-5)^{n-1}}$

$a_8 = 3(-5)^7 = \boxed{-234{,}375}$

$a_1 = -1$ $\boxed{r = 6}$

$\boxed{a_n = -1(6)^{n-1}}$

$a_8 = -1(6)^7 = \boxed{-279{,}936}$

⑥ $\sum_{k=1}^{5} (30 - k^2) = \boxed{95}$ $\sum_{m=1}^{5} (4m^2 + 4) = \boxed{240}$

⑦ $\sum_{a=4}^{9} (20 - a^2)$ $\boxed{\text{first} = 4 \\ \text{last} = -61}$

⑧ $\boxed{\sum_{x=1}^{4} 3^x}$ $\boxed{\sum_{x=1}^{4} 4x}$

⑨ $\boxed{\text{converges} \\ \text{sum} = 1.5}$ $\boxed{\text{converges} \\ \text{sum} = 2}$

⑩ $S_{100} = 27\left(\frac{1-(\frac{1}{3})^{100}}{1-(\frac{1}{3})}\right) = \boxed{40.5}$ $S_{10} = 40\left(\frac{1-(\frac{1}{2})^{10}}{1-(\frac{1}{2})}\right) = \boxed{79.921875 \\ \text{OR} \\ \frac{5115}{64}}$

⑪ $S = \frac{50}{1-0.99} = \boxed{5000\text{cm}}$ ⑫ $S_5 = 5500\left(\frac{1-1.2^5}{1-1.2}\right) = \boxed{40{,}928.8}$

⑬ $r = \sqrt{\frac{18.5}{20}} = 0.961769$

$20 \times = \boxed{19.235}$

$r = \sqrt{\frac{16.56}{46}} = 0.6$

$\times 46 = \boxed{27.6}$

① $\boxed{5^3 = 125}$ ② $\boxed{8^{1/3} = 2}$ ③ $\boxed{3^4 = 81}$ ④ $\boxed{50^0 = 1}$ ⑤ $\boxed{3^{-3} = \frac{1}{27}}$

⑱ $\sqrt{81} = \boxed{9}$ ⑲ $\boxed{6}$ ⑳ $\boxed{\frac{1}{2}}$ ㉑ $\boxed{-3}$ ㉒ $\sqrt[3]{-27}^2 = (-3)^2 = \boxed{9}$

㉓ $\boxed{-2}$ ㉔ $\boxed{2}$ ㉕ $\boxed{5}$ ㉖ $\boxed{-3}$ ㉗ $\boxed{0}$ ㉘ $\boxed{-2\frac{1}{3}}$

㉙ $\boxed{-1.5}$