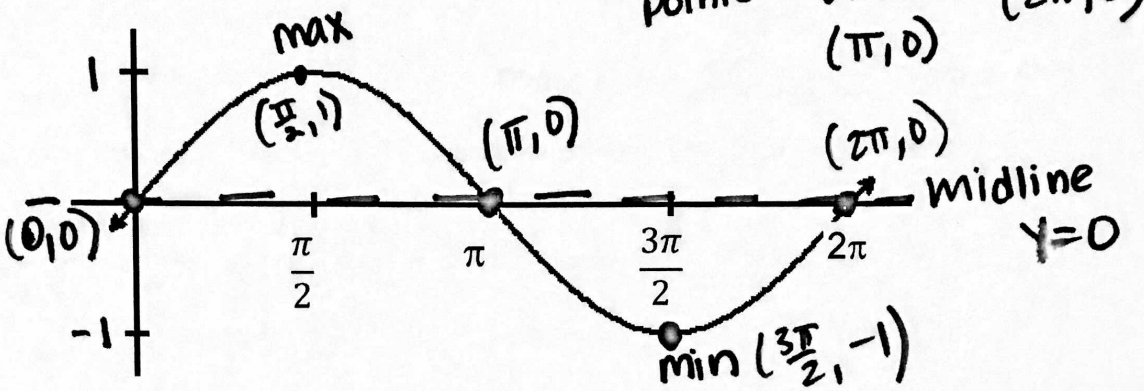
Day 11 Notes: Graphing Sine Functions

Critical points: $(0,0)$ $(\frac{\pi}{2},1)$ $(\pi,0)$ $(\frac{3\pi}{2},-1)$ $(2\pi,0)$

$y = \sin x$



Characteristics of a Sine Graph

Period - 1 period = 1 complete cycle (2π)

Domain - x-values (left to right) $\mathbb{R}$ $(-\infty,\infty)$

Range - y-values (bottom to top) $[-1,1]$

Amplitude - distance from midline to max or min

Phase Shift - how far to move left or right

Vertical Shift - how far to move up or down

Graphing Variations of the Sine Function

$$Y = a \sin [b(x - c)] + d$$

(+) left, (−) right (opposite)

(+) up, (−) down

a: Amplitude (+) Period: $\frac{2\pi}{b}$ Phase Shift: c (inside) Vertical Shift: d (outside)

1. Identify the amplitude and period.
2. Find the values of x for the five key points (the x-intercepts, the maximum point, and the minimum point). Start with the value of c where the cycle begins and add the quarter period.
3. Find the values of y for the 5 key points by evaluating the sine function

← phase shift

$X_1 = C$ where the cycle begins

x_2: $x_1 + \frac{\text{period}}{4}$ x_3: $x_2 + \frac{\text{period}}{4}$ x_4: $x_3 + \frac{\text{period}}{4}$ x_5: $x_4 + \frac{\text{period}}{4}$

4. Connect the 5 key points with a smooth curve and graph one complete cycle of the function.
5. Extend the graph to the left and right as desired.

EX.

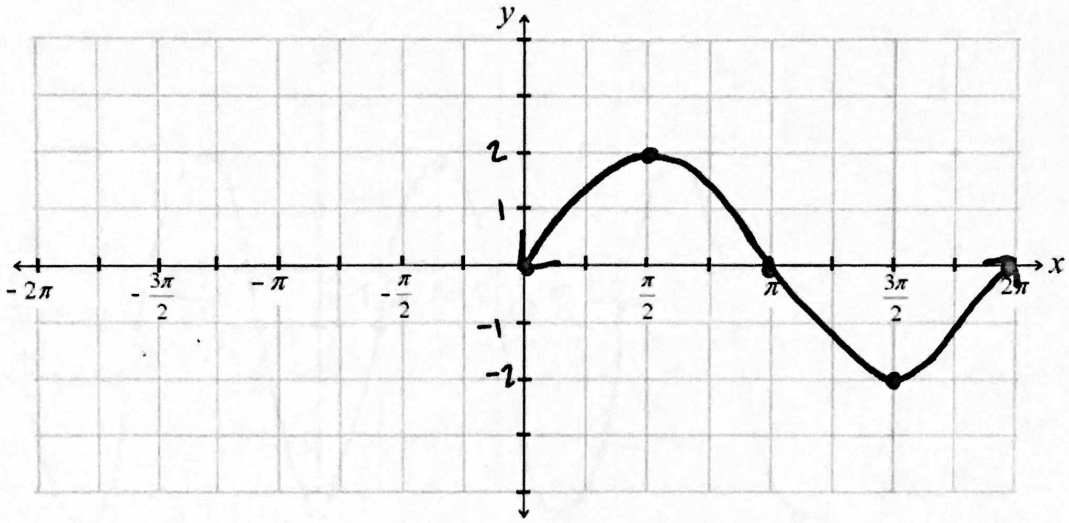
1. Graph $y = 2\sin x$

Amplitude = 2 Period = $\frac{2\pi}{1} = 2\pi$ Phase shift = none ($c=0$) Vertical Shift = none ($d=0$)

Find the 5 key points. Dive the period by 4 to get the quarter period. We start with the value of x where the cycle begins.

$x_1 = 0$ $x_2 = 0 + \frac{2\pi}{4} = \frac{\pi}{2}$ $x_3 = \frac{\pi}{2} + \frac{\pi}{2} = \pi$ $x_4 = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$ $x_5 = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

$y_1 = 0$ $y_2 = 2$ $y_3 = 0$ $y_4 = -2$ $y_5 = 0$



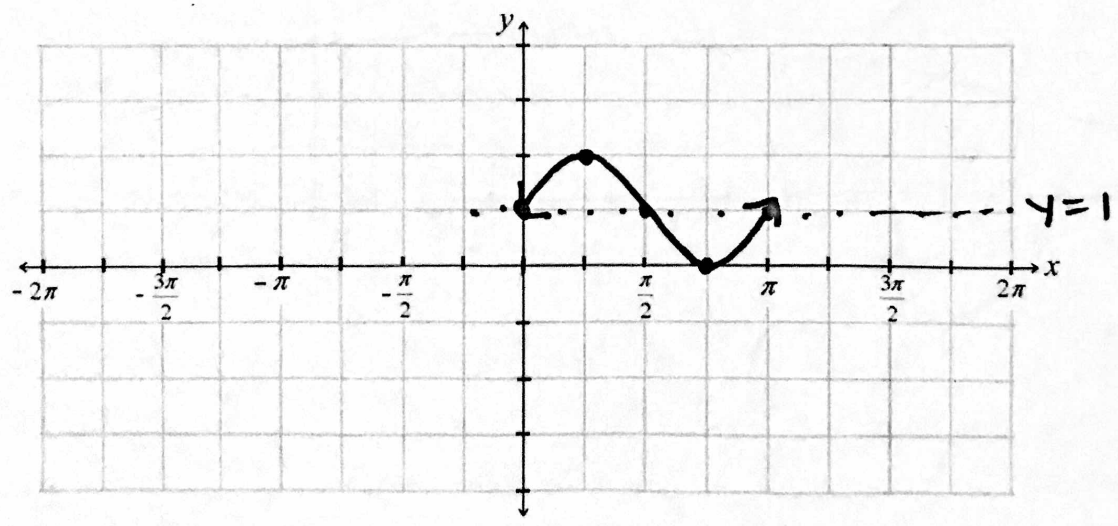
2. $y = \sin(2x) + 1$

Amplitude = 1 Period = $\frac{2\pi}{2} = \pi$ Phase shift = none ($c=0$) Vertical Shift = up 1

Find the 5 key points. Dive the period by 4 to get the quarter period. We start with the value of x where the cycle begins.

$x_1 = c = 0$ $x_2 = 0 + \frac{\pi}{4} = \frac{\pi}{4}$ $x_3 = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$ $x_4 = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$ $x_5 = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$y_1 = 0 + 1 = 1$ $y_2 = 2$ $y_3 = 1$ $y_4 = 0$ $y_5 = 1$



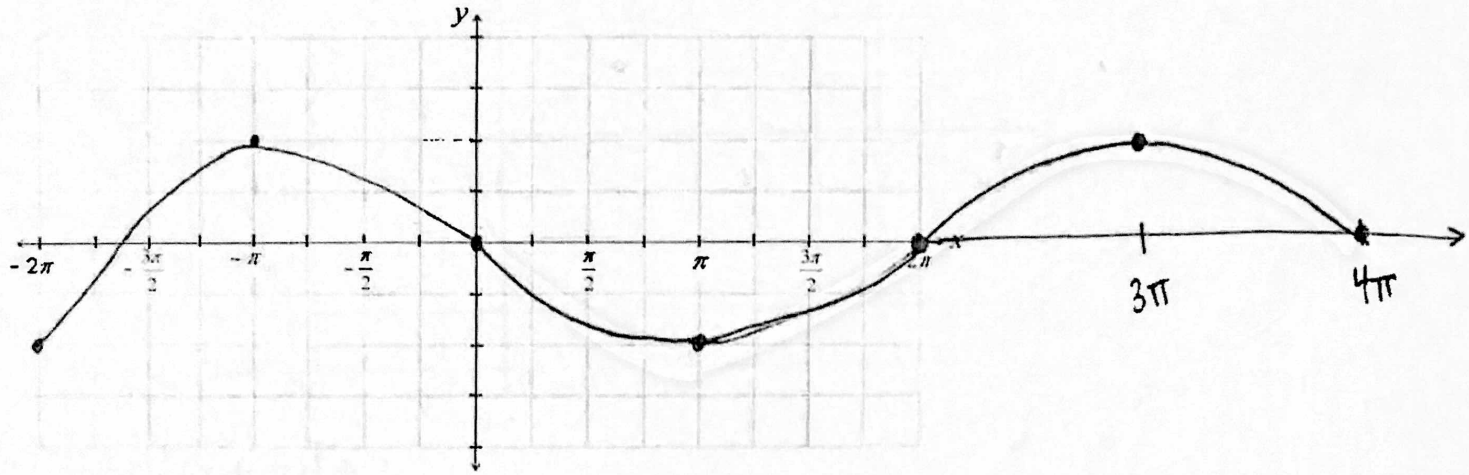
3. $y = -2\sin\left(\frac{x}{2}\right)$

Amplitude = 2 Period = $\frac{2\pi}{1/2} = 4\pi$ Phase shift = $\varnothing$ Vertical Shift = $\varnothing$

Find the 5 key points. Divide the period by 4 to get the quarter period. We start with the value of x where the cycle begins. → π

$x_1 = 0$ $x_2 = \pi$ $x_3 = 2\pi$ $x_4 = 3\pi$ $x_5 = 4\pi$

$y_1 = 0$ $y_2 = -2$ $y_3 = 0$ $y_4 = 2$ $y_5 = 0$



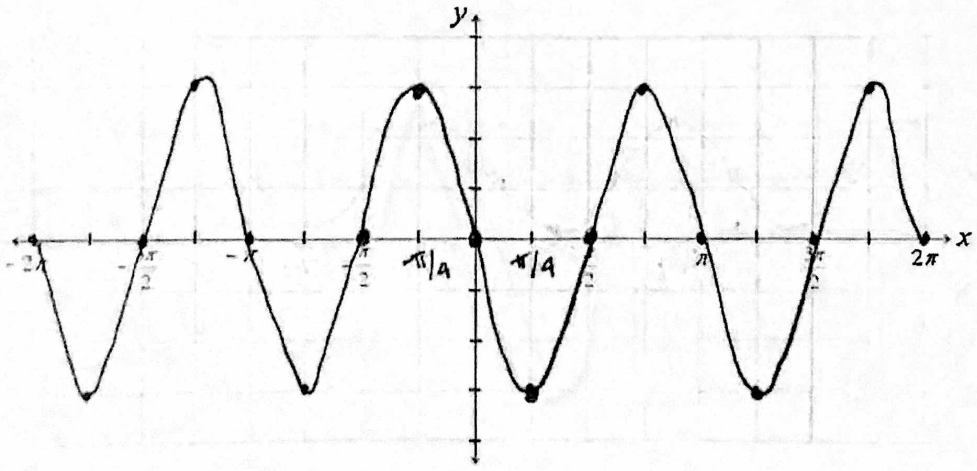
4. $y = 3\sin 2\left(x + \frac{\pi}{2}\right)$

Amplitude = 3 Period = π Phase shift = left $\frac{\pi}{2}$ Vertical Shift = $\varnothing$

Find the 5 key points. Divide the period by 4 to get the quarter period. We start with the value of x where the cycle begins. $\pi/4$

$x_1 = -\pi/2$ $x_2 = -\pi/4$ $x_3 = 0$ $x_4 = \pi/4$ $x_5 = \pi/2$

$y_1 = 0$ $y_2 = 3$ $y_3 = 0$ $y_4 = -3$ $y_5 = 0$



5.. $y = 3\sin\left[2\left(x - \frac{\pi}{4}\right)\right] - 1$

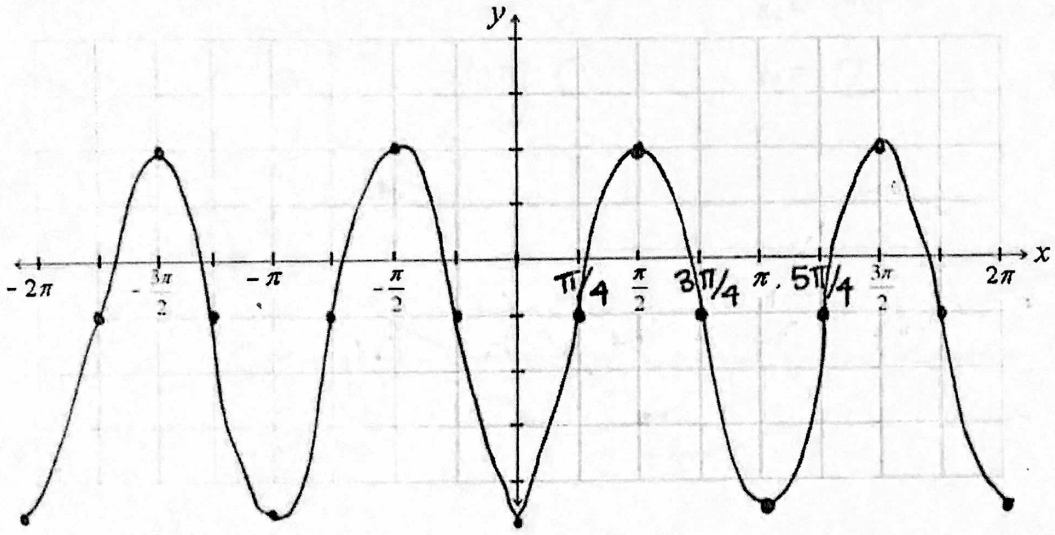
Amplitude = 3 Period = π Phase shift = right $\pi/4$ Vertical Shift = $\downarrow 1$

Find the 5 key points. ~~Dive~~ the period by 4 to get the quarter period. We start with the value of x where the cycle begins.

$\pi/4$

$x_1 = \pi/4$ $x_2 = 2\pi/4 = \pi/2$ $x_3 = 3\pi/4$ $x_4 = \pi$ $x_5 = 5\pi/4$

$y_1 = -1$ $y_2 = 2$ $y_3 = -1$ $y_4 = -4$ $y_5 = -1$



6. $y = \sin\left[\frac{1}{4}(x + \pi)\right] + 2$

Amplitude = 1 Period = $\frac{2\pi}{1/4} = 8\pi$ Phase shift = left π Vertical Shift = $\uparrow 2$

Find the 5 key points. Dive the period by 4 to get the quarter period. We start with the value of x where the cycle begins.

2π

$x_1 = -\pi$ $x_2 = \pi$ $x_3 = 3\pi$ $x_4 = 5\pi$ $x_5 = 7\pi$

$y_1 = 2$ $y_2 = 3$ $y_3 = 2$ $y_4 = 1$ $y_5 = 2$