Finding the Inverse Algebraically (unit 5 day 2)

(ex) $f(x) = 2x + 3$, $f^{-1}(x) = ?$

$y = 2x + 3$

$x = 2y + 3$

$x - 3 = 2y$

$\frac{x-3}{2} = y$

$\boxed{\frac{x-3}{2} = f^{-1}(x)}$

Steps

1. Change $f(x)$ to y
2. Switch x and y
3. Get y by itself
4. Change y to $f^{-1}(x)$

(ex) $f(x) = \frac{3}{x+1}$

$y = \frac{3}{x+1}$

Domain: $(-\infty, -1) \cup (-1, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

$(y+1)x = \frac{3}{y+1}(y+1)$

$(y+1)x = 3$

$y + 1 = \frac{3}{x}$

$y = \frac{3}{x} - 1$

$\boxed{f^{-1}(x) = \frac{3}{x} - 1}$

D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, -1) \cup (-1, \infty)$

(ex) $f(x) = \sqrt{x+3} - 2$

D: $[-3, \infty)$

R: $[-2, \infty)$

$y = \sqrt{x+3} - 2$

$x = \sqrt{y+3} - 2$

$x+2 = \sqrt{y+3}$

$(x+2)^2 = (\sqrt{y+3})^2$

$(x+2)(x+2) = y+3$

$x^2+4x+4 = y+3$

$x^2+4x+1 = y$

$\boxed{f^{-1}(x) = x^2+4x+1}$

D: $[-2, \infty)$

R: $[-3, \infty)$

(ex) $f(x) = 4\sqrt[3]{x-5}$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

$y = 4\sqrt[3]{x-5}$

$x = 4\sqrt[3]{y-5}$

$\frac{x}{4} = \sqrt[3]{y-5}$

$\left(\frac{x}{4}\right)^3 = y-5$

$\left(\frac{x}{4}\right)^3 + 5 = y$

$\boxed{f^{-1}(x) = \frac{x^3}{64} + 5}$

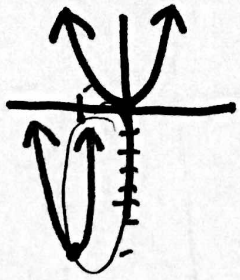
D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

(ex) $f(x) = 3(x+1)^2 - 7$

D: $[-1, \infty)$

R: $[-7, \infty)$



$y = 3(x+1)^2 - 7$

$x > 3(y+1)^2 - 7$

$\frac{x+7}{3} = (y+1)^2$

$\sqrt{\frac{x+7}{3}} = y+1$

$\sqrt{\frac{x+7}{3}} - 1 = y$

$\boxed{f^{-1}(x) = \sqrt{\frac{x+7}{3}} - 1}$

D: $[-7, \infty)$

R: $[-1, \infty)$

* (ex) $f(x) = \frac{x}{x-2}$

D: $(-\infty, 2) \cup (2, \infty)$

R: $(-\infty, 1) \cup (1, \infty)$

$y = \frac{x}{x-2}$

$(y-2)\ x = \frac{y}{y-2}(y-2)$

$(y-2)x = y$

$xy - 2x = y$ *

$xy = y + 2x$

$xy - y = 2x$ *factor out the y!

$y(x-1) = 2x$

$\boxed{y = \frac{2x}{x-1}}$

D: $(-\infty, 1) \cup (1, \infty)$

R: $(-\infty, 2) \cup (2, \infty)$