Unit 1
Day 7

Expected Value & Fair Game

- A **probability distribution** provides values of the random variable & its corresponding probability (usually a list or table)

    (1) Probabilities must add up to 1

    (2) All probabilities are between 0 and 1

outcomes	X_1	X_2	.....	X_n
probability	P_1	P_2		P_n

- **Expected value**: the mean (or avg) of all the probabilities in the distribution

→ what you should anticipate happening in the long run of many trials of the "game"

$$E(X) = (X_1)(P_1) + (X_2)(P_2) + \dots + (X_n)(P_n)$$

(ex) # of DVD's a person rents from a videostore during a single visit

possible outcomes

X	P(X)
0	.06
1	.58
2	.22
3	.10
4	.03
5	.01

- Prob Dist? (Adds up to 1?) Yes!

$$E(X) = (0)(.06) + 1(.58) + 2(.22) + (3)(.10) + (4)(.03) + (5)(.01) = \boxed{1.49}$$

the avg over time of the # of DVD's rented in one visit ex Health Insurance Policy

- Insurance Policy is \$250,000
- sold to woman for \$520
- one year
- Probability she survives: 0.99791

- EV? to Insurance Policy

	$\boxed{X}$	$\boxed{P}$
• She lives:	\$520	.99791
• She dies:	-\$249,480	.00209

$$E(x) = (520)(.99791) + (-249{,}480)(.00209)$$

$$= -2.5$$

overtime, the insurance company will ~~lose~~ lose \$2.50 on average w/ this policy

* A game is a fair game when the expected value of both participants is zero.

ex You pay \$3.00 to play. The dealer deals you one card from a standard deck. If it's a spade, you get \$10. If it's anything else, you lose your money.

• Spade	\$7.00	1/4
• anything else	-\$3.00	3/4

$$E(x) = (7)\left(\frac{1}{4}\right) + (-3)\left(\frac{3}{4}\right) = -.5$$

Not a fair game, bc not zero.

Fair Games $\boxed{\text{Key}}$

1. You pay \$3.00 to play. The dealer deals one card. If it is a spade, you get \$10. If it is anything else, you lose your money. Is this game fair?

$1(-3) + \frac{1}{4}(10) + \frac{3}{4}(0)$

$\boxed{\text{or}}$

Outcome :	\$7	-\$3
Probability:	$\frac{1}{4}$	$\frac{3}{4}$

$= 7(\frac{1}{4}) + (-3)(\frac{3}{4}) = \boxed{-.50}$ $\boxed{\text{NO}}$

Not fair

2. A casino game costs \$3.50 to play. You draw 1 card from a deck of standard cards. If it is a heart, you win \$10; if it is a Queen of hearts, you win \$50. Is this a fair game?

Q♡ 1/52	♡ 12/52	39/52
\$46.50	\$6.50	-3.50

$= -.23$ $\boxed{\text{NO}}$ Since the casino, on average, earns 23 cents more than it pays out to you, it's not a fair game.

3. A player rolls a die and receives the number of dollars equal to the number on the die EXCEPT when the die shows 6. If a 6 is rolled, the player loses \$6. If the game is fair, what should be the cost to play?

1	2	3	4	5	6
1	2	3	4	5	-6
1/6	1/6	1/6	1/6	1/6	1/6

$= \$1.50$ ← So charge \$1.50 to play

4. Consider the above game with a modification. We would like to make a fair, FREE game. We will do this by charging the customer money if they roll a 1 as well as a 6. If all the rest is the same, what would we charge if they roll a 1?

$(x)(\frac{1}{6}) + \underline{2(\frac{1}{6}) + 3(\frac{1}{6}) + 4(\frac{1}{6}) + 5(\frac{1}{6}) - 6(\frac{1}{6})} = 0$

$x(\frac{1}{6}) + \frac{4}{3} = 0$

$\frac{1}{6}x = -\frac{4}{3} \rightarrow x = -8$ so $\boxed{\text{Charge } \$8}$

5. This last game costs \$1 to play. You are given a coin to flip. If you flip tails, the game ends. If you flip heads you may flip again for a max of 5 flips. You will be paid \$1 for each head. If all 5 flips result in heads, you win the \$5 for 5 heads plus a \$2 bonus. Is the game fair?

T	HT	HHT	HHHT	HHHHT	HHHHH
-1	-1+1=0	-1+1=1	-1+3=2	-1+4=3	-1+5=4 +2=6
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$

$E(X) = .03$ so $\boxed{\text{not fair}}$

(but good for player!)