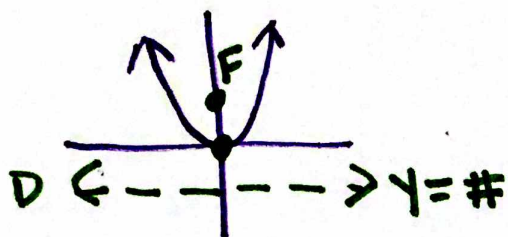


Equation of a Parabola with Focus & Directrix (Unit 3) (Day 9)

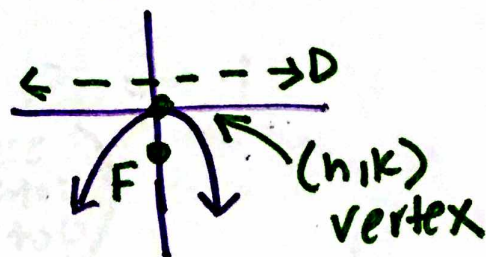
- A parabola is the set of all points that are the same distance from a fixed point (focus) and a fixed line (directrix)

vertex form

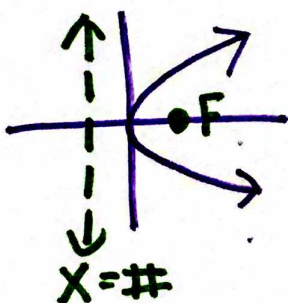
$$y = a(x-h)^2 + k$$



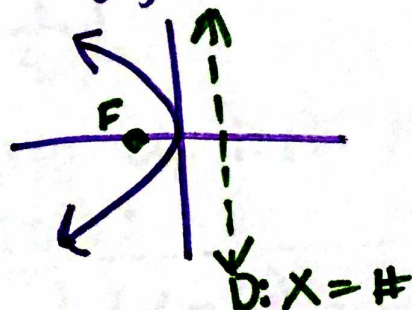
$$y = -a(x-h)^2 + k$$



$$x = a(y-k)^2 + h$$



$$x = -a(y-k)^2 + h$$



*"p" is the distance that the focus and the directrix are from the vertex

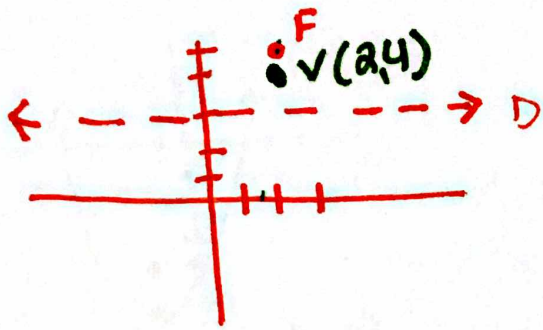
p = focus to vertex

= vertex ^{or} to directrix

$$p = \frac{1}{4a}$$

$$\text{or } a = \frac{1}{4p}$$

ex) write the equation for a parabola if the focus is at $(2, 5)$ and directrix is at $y = 3$



$$y = a(x-h)^2 + k \quad (h, k)?$$

a?

Steps

① Find the vertex
using graph: $(2, 4)$

② Find $p = 1$
(Distance from F to V or V to D)

③ solve for $a = \frac{1}{4(1)} = \frac{1}{4}$
(use $a = \frac{1}{4p}$)

④ substitute a, h, k into equation:

$$y = \frac{1}{4}(x-2)^2 + 4$$

ex) write eqn

Standard form:

$$y = \frac{1}{4}(x-2)^2 + 4$$

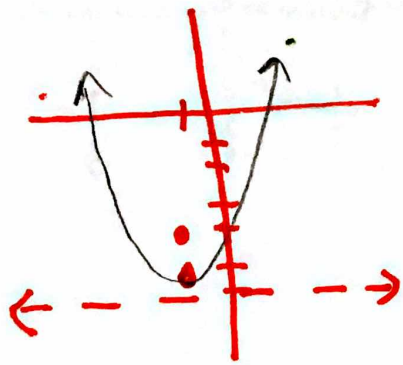
$$y = \frac{1}{4}(x-2)(x-2) + 4$$

$$y = \frac{1}{4}(x^2 - 4x + 4) + 4$$

$$y = \frac{1}{4}x^2 - x + 1 + 4$$

$$y = \frac{1}{4}x^2 - x + 5$$

ex) write the equation if focus is $(-1, -4)$
and directrix is $y = -6$



$$(h, k) = (-1, -5)$$

$$p = 1$$

$$a = \frac{1}{4p} = \frac{1}{4}$$

$$y = a(x-h)^2 + k \rightarrow \boxed{y = \frac{1}{4}(x+1)^2 - 5}$$

vertex form

Convert to standard form:

$$y = \frac{1}{4}(x+1)^2 - 5$$

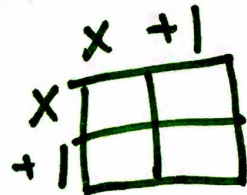
$$y = \frac{1}{4}(x+1)(x+1) - 5$$

$$y = \frac{1}{4}(x^2 + 2x + 1) - 5$$

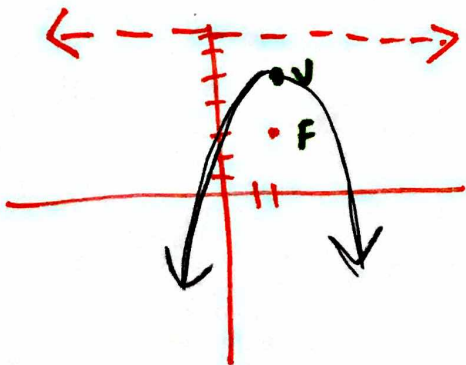
$$y = \frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4} - 5$$

$$\boxed{y = \frac{1}{4}x^2 + \frac{1}{2}x - \frac{19}{4}}$$

Standard Form



ex) write eqn: Focus $(2, 3)$ and Directrix: $y = 7$



$$(h, k) = (2, 5)$$

$$p = 2$$

$$a = \frac{1}{8}$$

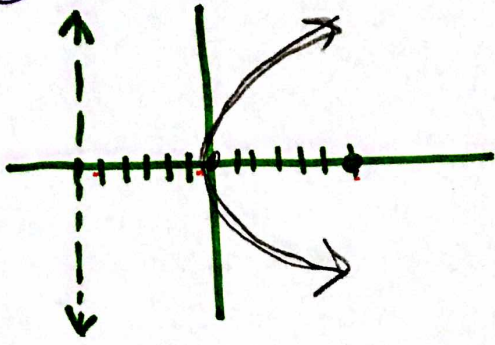
$$\boxed{y = -\frac{1}{8}(x-2)^2 + 5}$$

vertex Form

Standard Form:

$$\boxed{y = -\frac{1}{8}x^2 + \frac{1}{2}x + \frac{9}{2}}$$

ex) Write eqn: Focus $(6, 0)$ and $x = -6$ (Directrix)



$$(h, k) = (0, 0)$$

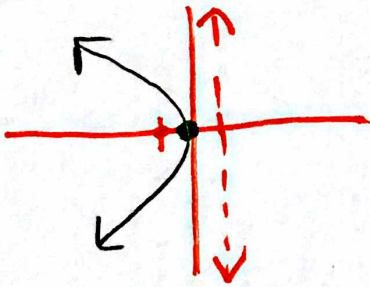
$$p = 6$$

$$a = \frac{1}{24}$$

$$x = \frac{1}{24}(y-0)^2 + 0$$

$$x = \frac{1}{24}y^2$$

ex) write eqn: Focus $(-1, 0)$ and $x = 1$ (Directrix)



$$(h, k) = (0, 0)$$

$$p = -1$$

$$a = \frac{1}{4}$$

$$x = -\frac{1}{4}(y-0)^2 + 0$$

$$x = -\frac{1}{4}y^2$$