Key

M3H U4 Day 12 - Sequences and Series Applications

1. In 1970, a family membership in a country club cost $650. This cost increased $35 each year since 1970. Write a rule for the general term of the Arithmetic Sequence. Then use the rule to find out how much a membership in 1990 cost?

Note

$y = 650 + 35x$

note: 1970 → x=0

$x = 20 \rightarrow \boxed{y = \$1350}$

$y = 650 + 35(n-1)$

1970: n=1 so plug in 21 for 1990 cost

$= \boxed{1350}$

2. A department store chain had 112 franchises in 1978 and opened 8 new franchises each year thereafter. How many franchises were there at the end of 1985?

$y = 112 + 8x$

$x = 7 \rightarrow y = \boxed{168 \text{ stores}}$

3. The balcony of a theater has 12 rows of seats. The last row contains 8 seats, and each of the other rows contains one more seat than the row behind it. How many seats are there in the balcony?

8, 9, 10, ...

$a_n = 8 + 1(n-1)$

$a_{12} = 8 + 1(12-1) = 19$

$S_{12} = \frac{12}{2}(8+19)$

$= \boxed{162}$

4. Southwest Airlines' passenger load has been increasing by 12 percent annually. In 1980, they carried 20,500 passengers. Write a rule for this scenario then use the rule to determine how many passengers they expected to carry in 1990?

$y = 20{,}500(1.12)^x$

$y = \boxed{63{,}669}$ passengers

5. A pipe driver is used to drive a pipe into the ground. The first hit drives the pipe down 20 cm. If each succeeding blow drives the pipe down 0.3 times as far as the preceding hit, how far has the pipe been driven after 4 hits?

$20 + 6 + 1.8 + .54 = \boxed{28.34 \text{ cm}}$

$S_4 = \frac{20(1-.3^4)}{1-.3} = \boxed{28.34}$

6. An investment in a gas well earned a total of $54,450 during the first 5 yrs. Each year after the first, the investment earned 3 times as much as during the preceding year. How much did the investment earn during the first year?

$S_n = \frac{a_1(1-r^n)}{1-r} \rightarrow 54{,}450 = \frac{a_1(1-3^5)}{1-3}$

$\boxed{a_1 = 450}$

7. The end of the pendulum of a clock travels 40 cm on its first swing. On each succeeding swing, it travels 0.8 as far as it did on the preceding one. If it continues in this manner, how far will the end of the pendulum travel?

$40 + 32 + 25.6, \ldots$

$S_\infty = \frac{40}{1-.8} = \boxed{200 \text{ cm}}$

8. The pointer of a broken metronome travels 20 in. on its first swing. On each succeeding swing it travels 7/8 as far as it did on the preceding one. If it continues in this manner, how far will the pointer travel in all?

$S_\infty = \frac{20}{1-\frac{7}{8}} = \boxed{160 \text{ cm}}$

Part 2

1) A gardener planted a new type of ornamental grass and kept a record of its height over the first 14 days. How much does the grass grow each day? $.7$ cm
Write an explicit formula that gives the height of the grass after n days: $H = 4.2 + .7n$
How long will it take for the grass to be 28 cm tall? 34 days

Time (days)	0	3	7	10	14
Height (cm)	4.2	6.3	9.1	11.2	14

$\frac{6.3}{4.2} = 2.1 \rightarrow \frac{2.1}{3} = .7$ each day

$H = 4.2 + .7n$

$28 = 4.2 + .7n$

$n = 34$

2) Suppose an antique car you own appreciates (increases) in value at a rate of 8 percent annually. If the car was purchased for $12,000, what will it be worth during the eighth year?

$y = 12{,}000(1.08)^x$

$y = 12{,}000(1.08)^8 = \boxed{\$22{,}211.16}$

3) A 16-in. piece of wire is cut in half. Each piece is then cut in half, and the process continues. How many pieces are there after the 10th cutting of the pieces? What is the length of each piece after the 6th cutting?

- Pieces: $y = 1(2)^{10} = \boxed{1024}$ pieces
- Length: $y = 16\left(\frac{1}{2}\right)^6 = \boxed{.25 \text{ inches}}$

4) A contractor agreed that if a job was not done by a certain date, he would pay a $1000 penalty for the first day of delay, and for each day after that, he would pay $50 more than for the preceding day. How many penalty days did he use if his penalty was $10,800?

$a_n = 1000 + 50(n-1)$

$1000 + 50n - 50$

$950 + 50n$

$10{,}800 = \frac{n}{2}(a_1 + a_n)$

$10{,}800 = \frac{n}{2}(1000 + 950 + 50n)$

$10{,}800 = \frac{n}{2}(1950 + 50n)$

$0 = 25n^2 + 975n + 10800$

$10{,}800 = 975n + 25n^2 \rightarrow$

$n =$ omit

5) Find the sum of the integers from 1 through 100.

$S_{100} = \frac{100}{2}(1 + 100) = 5050$

6) John earned $50, 000 during the first year of his job at city hall. After each year he received a 10% raise. Find his total earnings during the first seven years on the job.

$\sum_{x=1}^{7} 50{,}000(1.10)^{x-1}$ or $S_7 = \frac{50{,}000(1 - 1.1^7)}{1 - 1.1}$

$\boxed{S_7 = \$474{,}358.55}$

7) The end of a spring is pulled as far as it will go and then is released. On the first bounce back it extends 32 cm. On its second bounce back it extends 16 cm. On its third bounce back it extends 8 cm. How many times will the spring bounce back before it extends 1 cm?

$1 = 32\left(\frac{1}{2}\right)^{n-1}$

$\boxed{n = 5 \text{ times}}$

8) A club sponsor needs to contact all of the club members by telephone to inform them of a change in plans. The sponsor calls 2 members, each of whom calls 2 other members not previously called, and so on. If all members have been contacted after the process has been repeated 6 times, how many members (excluding the sponsor) are in the club?

$\sum_{n=1}^{6} 2^x$ or $S_6 = \frac{2(1 - 2^6)}{1 - 2} = \boxed{126}$ members

9) After its release, a weather balloon rises 100 ft in the first minute. Each minute thereafter, the balloon rises only 80 percent as far as in the previous minute. What is the maximum height the balloon will reach?

$r = .8$

$|.8| < 1$ so converges to: $S_\infty = \frac{100}{1 - .8} = \boxed{500 \text{ ft}}$