

Completing the square

(ex) $x^2 - 12x + 5 = 0$

$$x^2 - 12x = -5$$

$$x^2 - 12x + \underline{36} = -5 + \underline{36}$$

$$(x-6)(x-6) = 31$$

$$(x-6)^2 = 31$$

$$\sqrt{(x-6)^2} = \sqrt{31}$$

$$x-6 = \pm\sqrt{31}$$

$$x = 6 \pm \sqrt{31}$$

*magic formula is

$$\left(\frac{b}{a}\right)^2$$

$$\left(\frac{-12}{1}\right)^2 = 36$$

Steps

- ① a must equal 1 (if not, factor it out)
- ② move constant to other side
- ③ leave room for magic # on both sides
- ④ Find magic # using $\left(\frac{b}{a}\right)^2$ and add in
- ⑤ Factor left side
- ⑥ solve

$$\textcircled{1} \quad x^2 + 6x + 10 = 0$$

$$x^2 + 6x + \underline{9} = -10 + \underline{9}$$

$$(x+3)^2 = -1$$

$$\sqrt{(x+3)^2} = \sqrt{-1}$$

$$x+3 = \pm i$$

$$\boxed{x = -3 \pm i}$$

$$\textcircled{\text{ex}} \quad 5x^2 - 6x - 8 = 0$$

$$5\left(x^2 - \frac{6}{5}x - \frac{8}{5}\right) = 0$$

~~5=0~~

$$x^2 - \frac{6}{5}x + \frac{9}{25} = \frac{8}{5} + \frac{9}{25}$$

$$\left(\frac{-6/5}{2}\right)^2$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{49}{25}$$

$$x - \frac{3}{5} = \pm \sqrt{\frac{49}{25}}$$

$$x - \frac{3}{5} = \pm \frac{7}{5}$$

$$x = \frac{3}{5} \pm \frac{7}{5}$$

$$x = \frac{3}{5} + \frac{7}{5} \quad \frac{3}{5} - \frac{7}{5} = \boxed{\frac{-4}{5}}$$

$$= \frac{10}{5} = \boxed{2}$$

$$3x^2 - 12x + 7 = 0$$

$$3(x^2 - 4x + \frac{7}{3}) = 0$$

$$\downarrow$$
$$\cancel{3 \neq 0}$$

$$\downarrow$$
$$x^2 - 4x + \frac{7}{3} = 0$$

$$x^2 - 4x + \underline{4} = -\frac{7}{3} + \underline{4}$$

$$(x-2)^2 = \frac{5}{3}$$

$$x-2 = \pm \sqrt{\frac{5}{3}}$$

$$x = 2 \pm \sqrt{\frac{5}{3}}$$

$$\rightarrow \begin{aligned} x &= 2 + \sqrt{\frac{5}{3}} \\ x &= 2 - \sqrt{\frac{5}{3}} \end{aligned}$$

Standard Form to vertex Form

- ① Move the constant to side of y and make space for magic number
- ② Factor out ~~get~~ a-value
- ③ Complete the square & get y by itself

ex) $y = x^2 + 10x - 3$

$$y + 3 = x^2 + 10x$$

$$y + 3 + \underline{25} = x^2 + 10x + \underline{25}$$

$$y + 28 = (x+5)^2$$

$$\boxed{y = (x+5)^2 - 28}$$

$$\boxed{\text{vertex: } (-5, -28)}$$

$$\left(\frac{10}{2}\right)^2 = 25$$

$$\text{ex) } y = 2x^2 - 8x + 5$$

$$y - 5 = 2x^2 - 8x$$

$$y - 5 = 2(x^2 - 4x)$$

$$\star \text{!! } y - 5 + \underline{8} = 2(x^2 - 4x + \underline{4})$$

$$y + 3 = 2(x - 2)^2$$

$$\boxed{y = 2(x - 2)^2 - 3}$$

$$\left(\frac{-4}{2}\right)^2 = 4$$